



Solver Choice in the SEM: LR vs. MIP

1st March 2011

➤ Why?

- Why optimisation software is needed;
- Short history of how these methods were developed:

➤ How?

- A look at how LR and MIP work;
- A review of Unit Commitment;

➤ What?

- Review of the findings of the LR vs MIP study

➤ What next?

- Consultation questions

➤ DISCLAIMER

- While we intend to give an **overview** of
 - Optimisation,
 - Lagrangian Relaxation, and
 - Mixed Integer Programming.
- This workshop is **not** intended to teach how these methods work;
- At the end of today, we hope you have a better idea of the techniques;
- And how they have been implemented in the SEM;
- The intention of today is to allow attendees understand the implications of what was done in the LR vs MIP study and contribute to the consultation phase.

➤ DISCLAIMER

- ABB's LR software is proprietary software.
- We cannot reveal its internal workings, nor do we fully know them.
- This is also true of IBM's ILOG CPLEX software.
- This is the MIP engine used by the ABB platform.
- IBM offer training in ILOG CPLEX through their website

Why?



- Why?
 - Because problems are difficult!
 - One approach is “brute force” enumeration
 - That is, to test every possible combination.
 - If we have one decision to make in four time intervals, how many combinations are available?

Solver Choice in the SEM: LR vs. MIP

Time intervals

Off in all four

On in all four

On in only one interval

On in two intervals

On in three intervals

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>
0	0	0	0
1	1	1	1
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
1	1	1	0
1	0	1	1
0	1	1	1
1	1	0	1

Solver Choice in the SEM: LR vs. MIP

- There are 16 combinations available in this example.
- If this was a commitment question, for one Generator in four Trading Periods, then 16 possible commitment outcomes.
- If there were two Generators, then there are 16 x 16 combinations.
- The formula for this is –

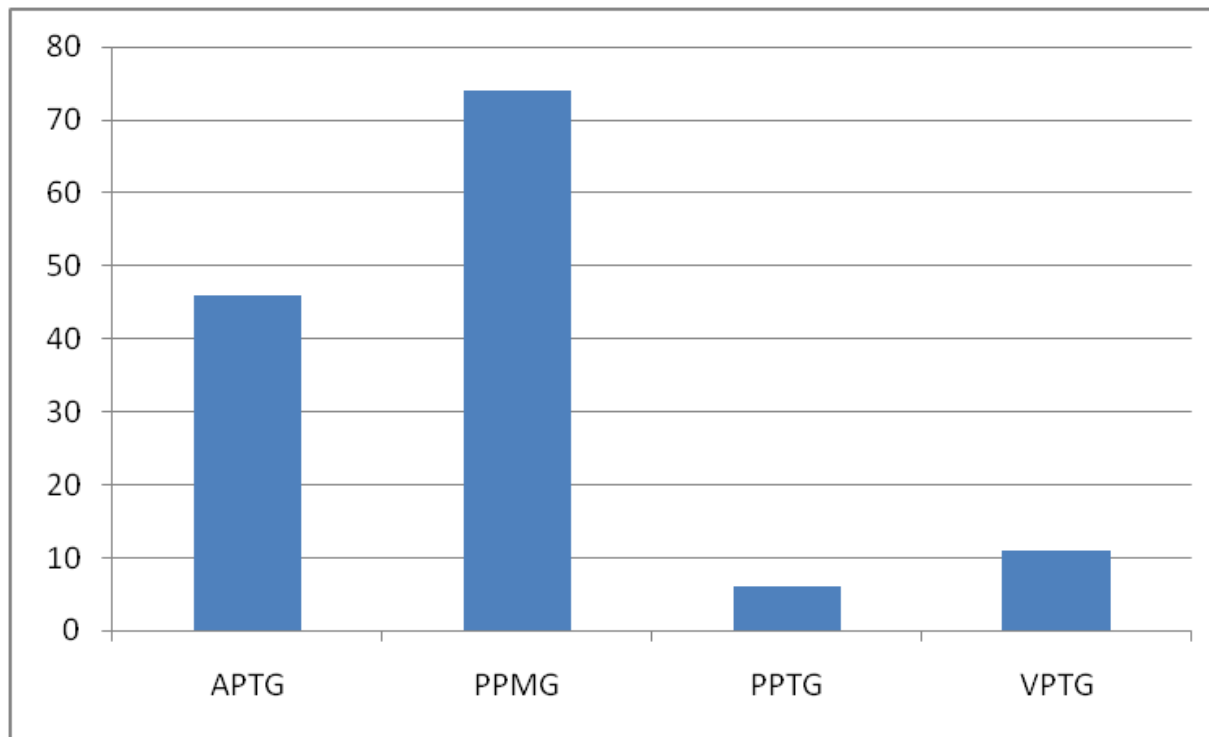
$$(2^n - 1)^m$$

- Where n is number of generators, and m is number of time intervals

- What is Unit Commitment?
 - The decision to have a power stations turned on or off;
 - Output of Market Scheduling and Pricing (MSP) software;
 - Decision based on balance requirement and economic merit order;
 - Requirement is to find the best or optimal solution for the problem;

Solver Choice in the SEM: LR vs. MIP

➤ And that's for **four** Generators!



➤ The SEM has 74...

- Our estimate of the number of combinations for 74 Generators over 60 Trading Periods ...

$$3.48 \times 10^{1338}$$

- If we could assess 1 billion combinations per second, then using brute force enumeration, by now we could have assessed ...

$$4.37 \times 10^{26}$$

- ...if we'd started just after the Big Bang.

- And that's why we need optimisation techniques.

How?



➤ Development of Optimisation

- Optimisation is the mathematical process of finding close to optimal solutions without the need of assessing all possible combinations;
- Makes it possible to solve large problems in realistic time periods;
- Modern methods developed during 1940's in the field of "Operations Research";
- Origins in the Second World War when British military asked scientists to analyse military problems

➤ Development of Optimisation

- George Bernard Dantzig considered the “father” of modern optimisation techniques;
- Dantzig credited with invention of linear programming (**LP**);
- Developed the Simplex algorithm;
- However, LP still requires that problems are *convex*;

➤ Example

<i>Ingredient</i>	<i>Stock Available</i>	<i>Qty in Product A</i>	<i>Qty in Product B</i>
<i>Ing - A</i>	1,400	4	4
<i>Ing - B</i>	1,800	6	3
<i>Ing - C</i>	1,800	2	6
<i>Profit</i>		€12	€8

➤ Problem is to maximise profit subject to available resources.

➤ Maximise $P = 12A + 8B$, subject to constraints -

➤ $4A + 4B \leq 1,400$

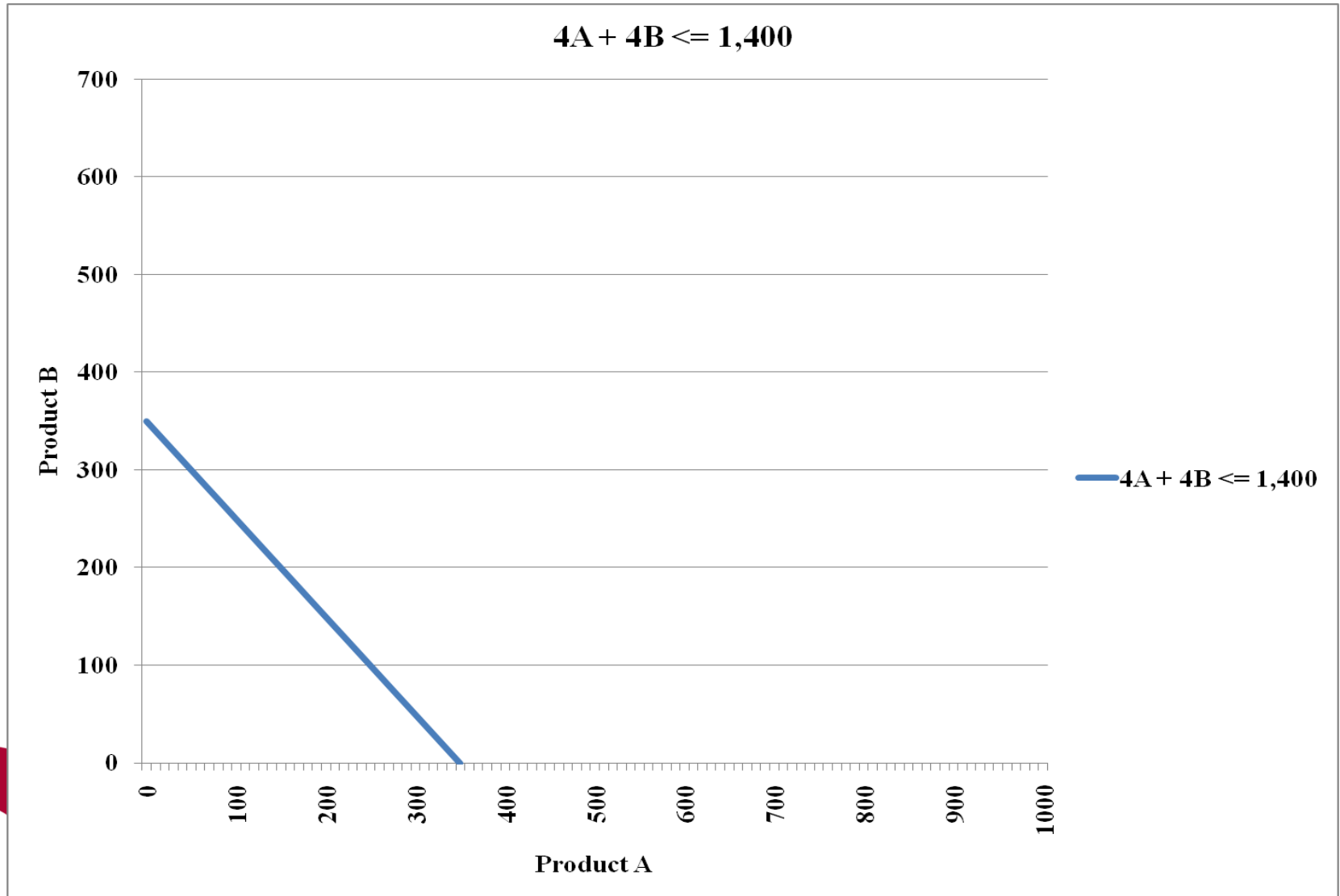
➤ $6A + 3B \leq 1,800$

➤ $2A + 6B \leq 1,800$

Solver Choice in the SEM: LR vs. MIP

- We can plot these constraints on a 2D graph.
- If $4A + 4B \leq 1,400$, then
- If $B = 0$, maximum value of $4A$ must not exceed 1,400
- Then, $A = 1,400/4$ (350)
- Equally, if $A = 0$, then maximum value of $4B$ must not exceed 1,400
- Then, $B = 1,400/4$ (350)
- Therefore, we can plot the constraint $4A + 4B \leq 1,400$ as a line from 350 on a y axis to 350 on an x axis as follows -

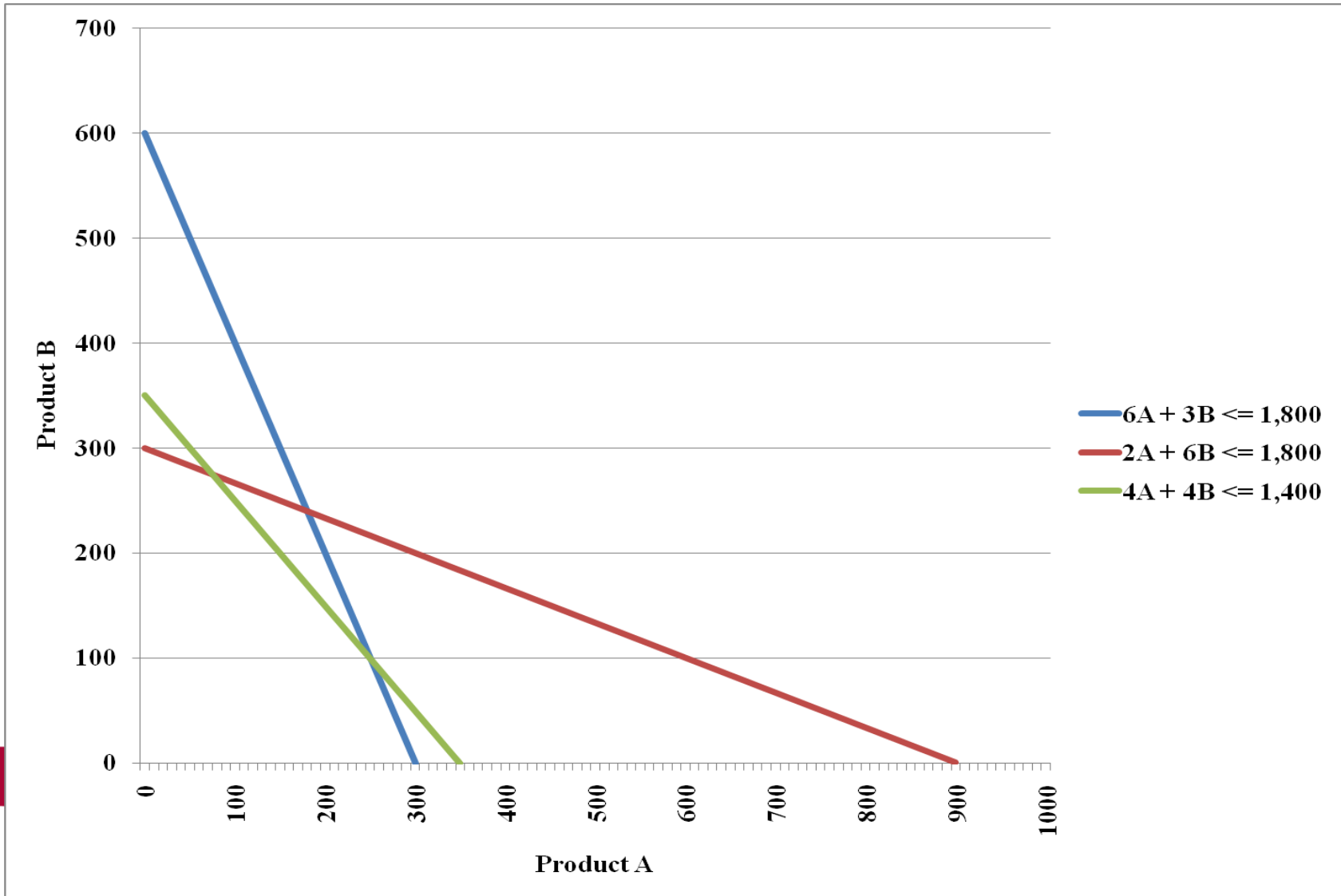
Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

- We can apply the same technique to the other constraints
- We get a line from 600 to 300 for $6A + 3B \leq 1,800$
- We get a line from 300 to 900 for $2A + 6B \leq 1,800$

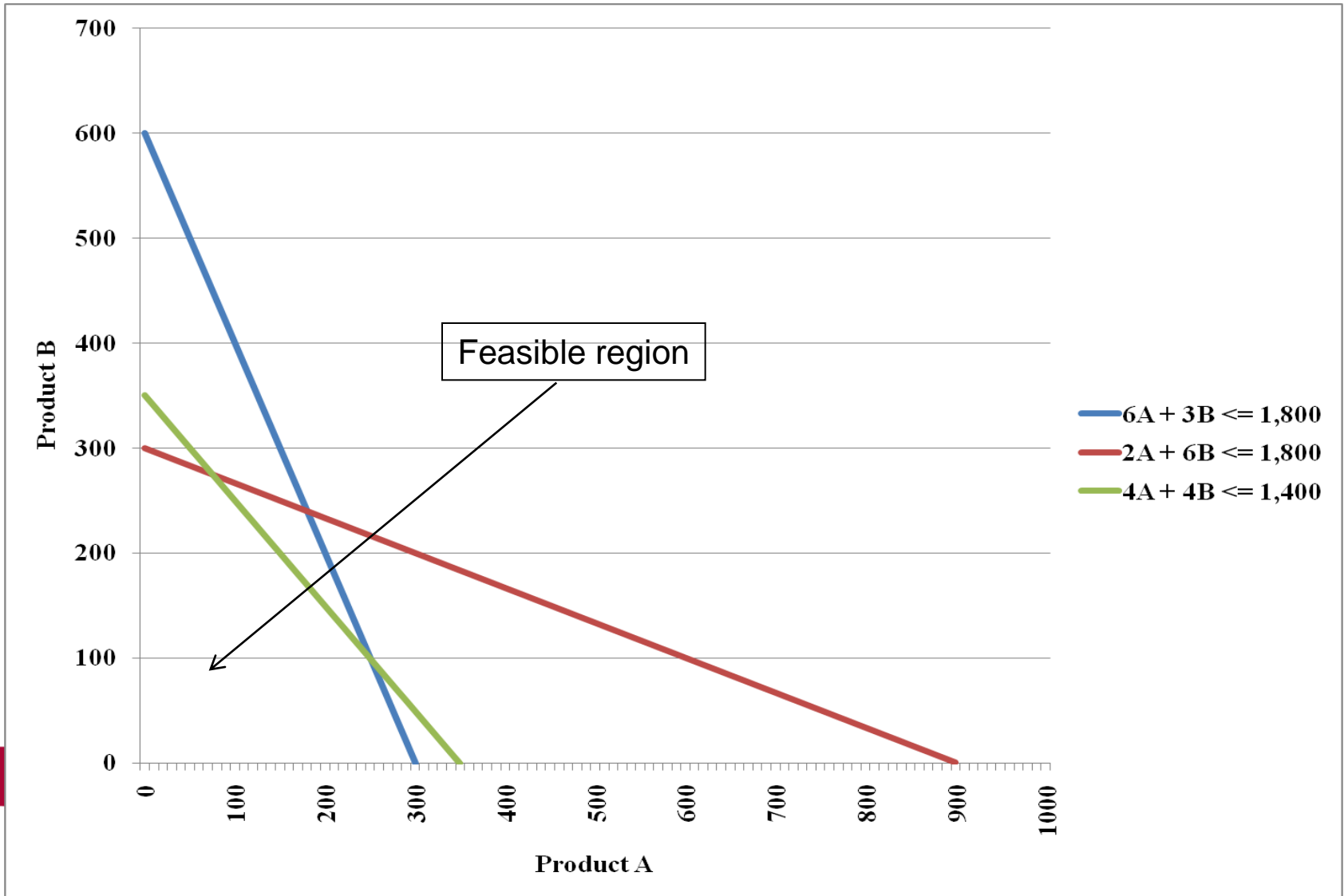
Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

- We can apply the same technique to the other constraints
- We get a line from 600 to 300 for $6A + 3B \leq 1,800$
- We get a line from 300 to 900 for $2A + 6B \leq 1,800$
- The area between the x and y axis and the three constraint lines is the feasible region
- All feasible solutions exist only within this area

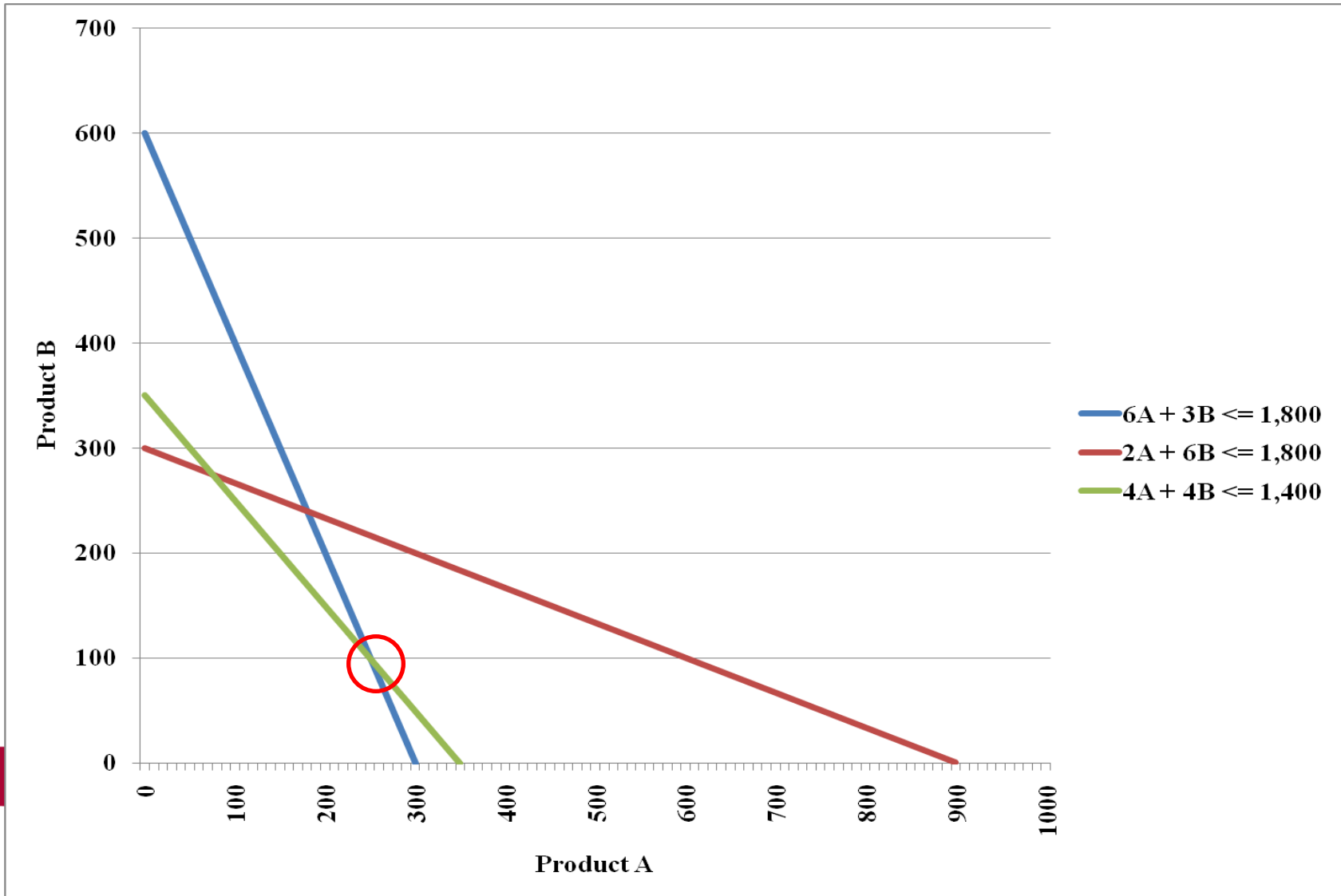
Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

- We can apply the same technique to the other constraints
- We get a line from 300 to 900 for $2A + 6B \leq 1,800$
- We get a line from 300 to 900 for $2A + 6B \leq 1,800$
- The area between the x and y axis and the three constraint lines is the feasible region
- All feasible solutions exist only within this area
- In this example, solution occurs at vertex or corner points

Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

- The simplex algorithm developed to solve problems like this;
- Done through the introduction of *slack variables*
- Allows us change the constraint from an inequality to an equality constraint
- That means,

- $4A + 4B \leq 1,400$

- $6A + 3B \leq 1,800$

- $2A + 6B \leq 1,800$

- can become

- $4A + 4B + S1 = 1,400$

- $6A + 3B + S2 = 1,800$

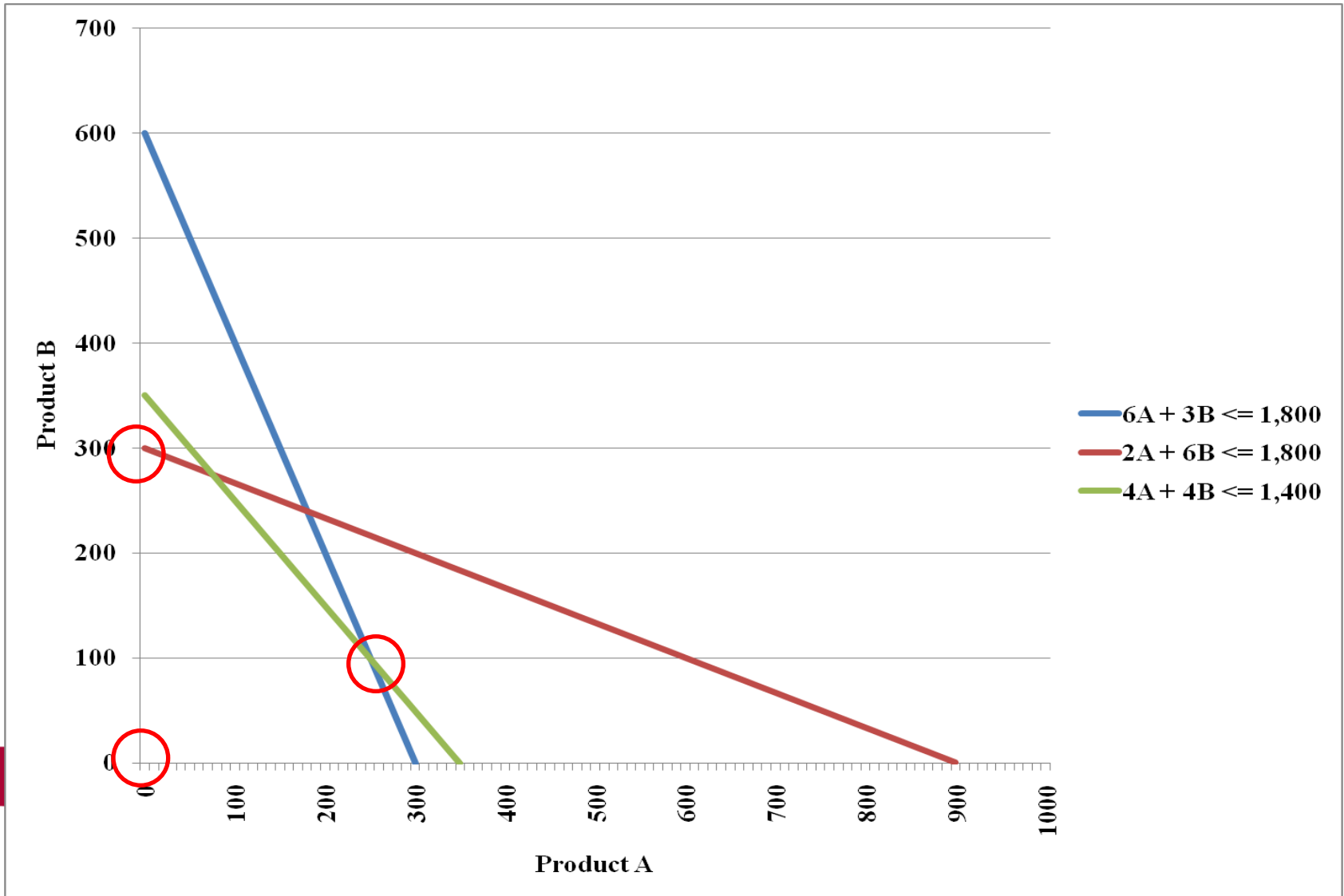
- $2A + 6B + S3 = 1,800$

Solver Choice in the SEM: LR vs. MIP

- The problem can be solved by selecting maximum feasible values for the variables in the original problem (Maximise $P = 12A + 8B$)
- If the problem is a Maximise, pick the larger coefficient
- If the problem is a Minimise, pick the smaller
- This will lead to the optimal solution



Solver Choice in the SEM: LR vs. MIP



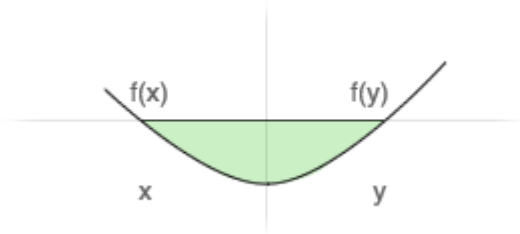
Solver Choice in the SEM: LR vs. MIP

- The problem can be solved by selecting maximum feasible values for the variables in the original problem (Maximise $P = 12A + 8B$)
- If the problem is a Maximise, pick the larger coefficient
- If the problem is a Minimise, pick the smaller
- This will lead to the optimal solution
- This will deliver an optimal solution when integer decisions not required

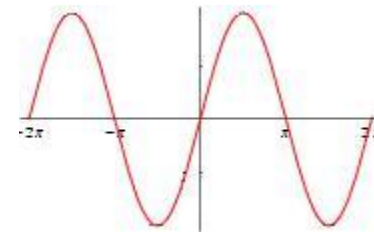
Solver Choice in the SEM: LR vs. MIP

- **LP** is a well established method for solving problems of the form -
- Maximise (or minimise) the cost associated with resources used, where this cost is a linear (straight-line) equation;
- Subject to various constraints;
- It can solve very large problems, very precisely;
- Shadow Price in the SEM is determined by an LP from the constraint requiring that demand and generation match;
- As noted, the problem must be *convex*.
- Therefore, linear programming is not suitable for Unit Commitment

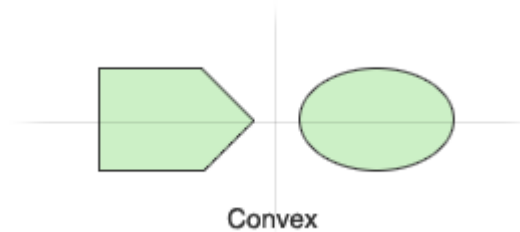
➤ *Convex vs. Non-Convex*



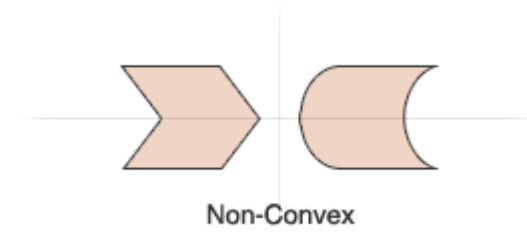
Vs.



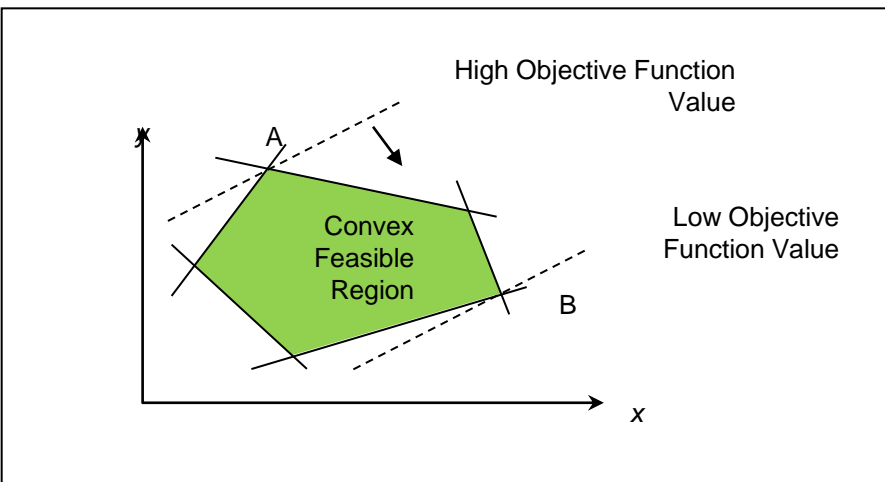
or



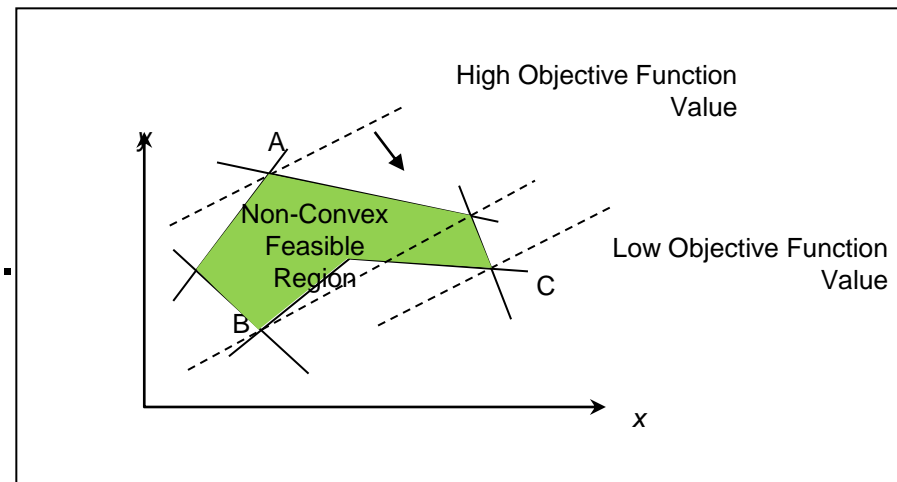
Vs.



➤ *Convex vs. Non-Convex*, in terms of feasible regions -



Vs.

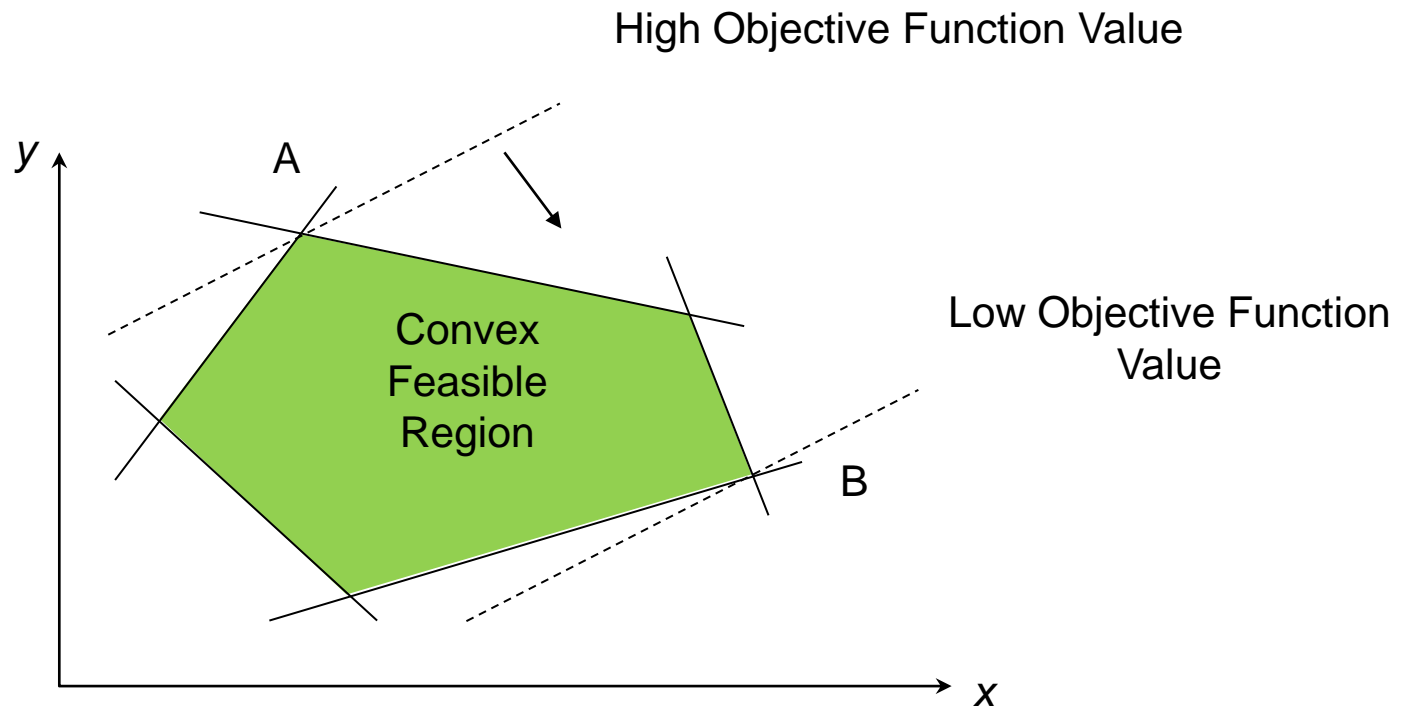


Solver Choice in the SEM: LR vs. MIP

- If the feasible region is convex, then it is possible to draw a line between any two possible solutions without that line leaving the set of possible solutions.



Solver Choice in the SEM: LR vs. MIP

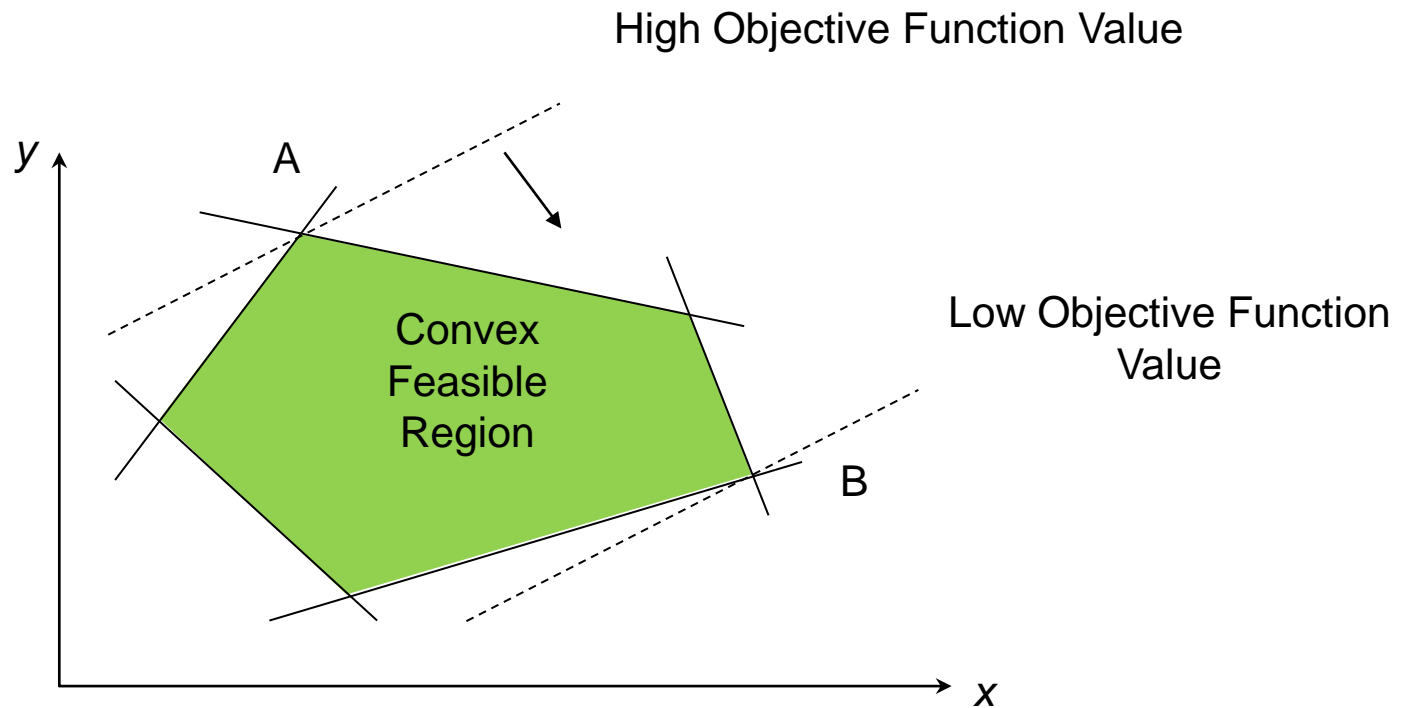


Solver Choice in the SEM: LR vs. MIP

- If the feasible region is convex, then it is possible to draw a line between any two possible solutions without that line leaving the set of possible solutions;
- A convex feasible region means that optimisation algorithms can always move towards better solutions and never get stuck;
- In the diagram, a cost minimisation problem is shown. The objective function has its highest value at point A and its lowest (optimal) value at point B. The algorithm can move around the corners of the feasible region from A to find the solution at B.



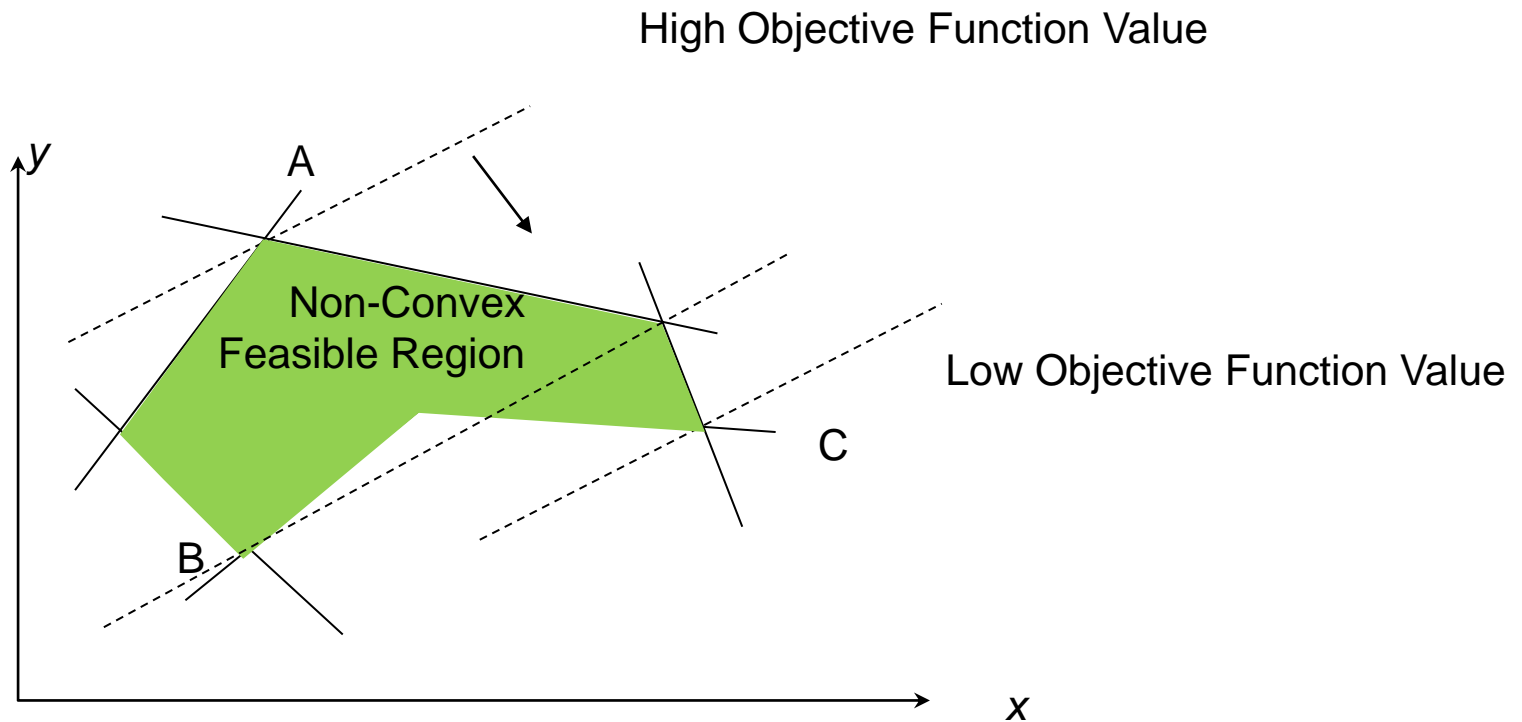
Solver Choice in the SEM: LR vs. MIP



- If the feasible region is non-convex, then it is not possible to draw a line between any two possible solutions without that line leaving the set of possible solutions;



Solver Choice in the SEM: LR vs. MIP

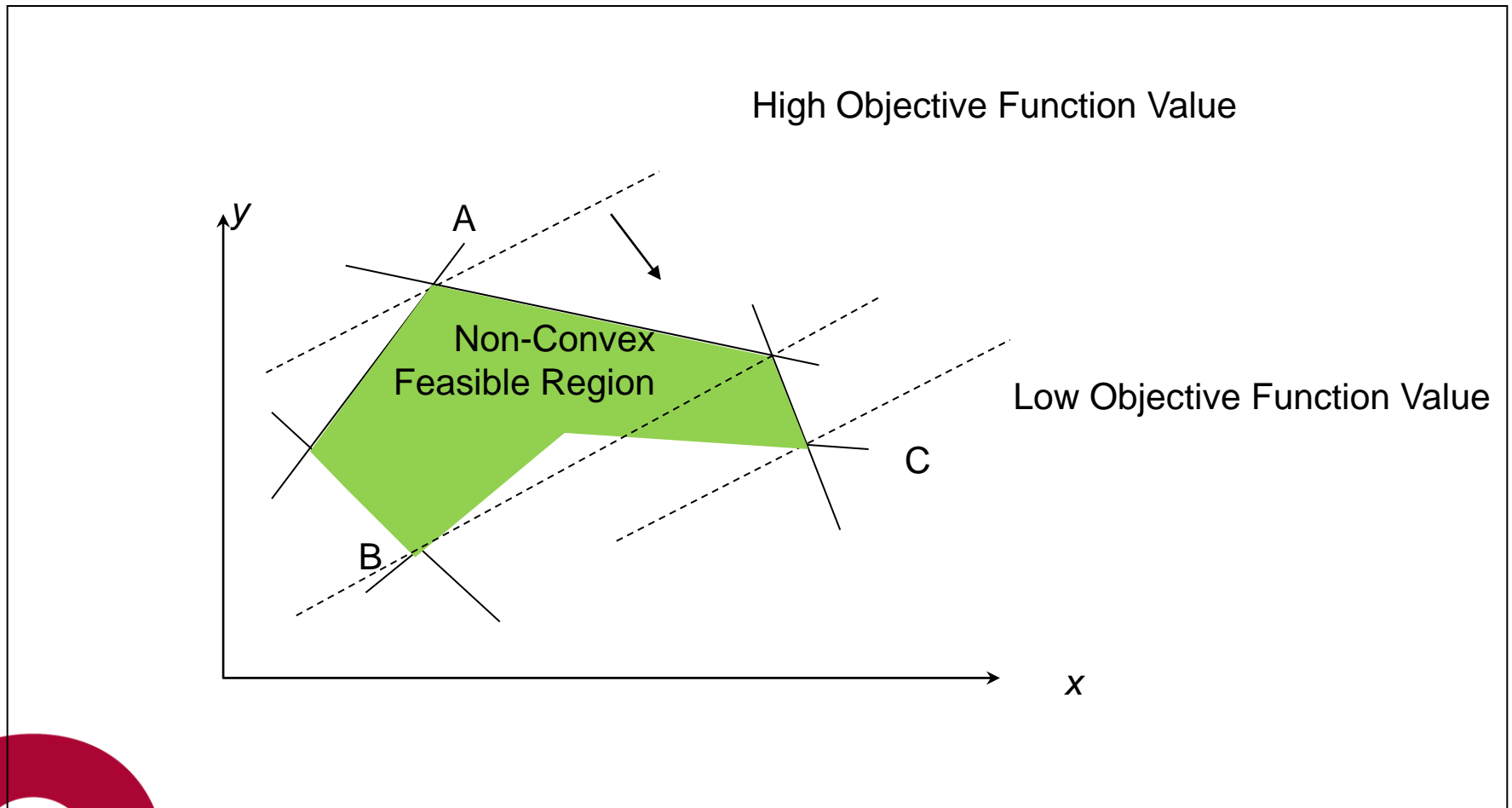


Solver Choice in the SEM: LR vs. MIP

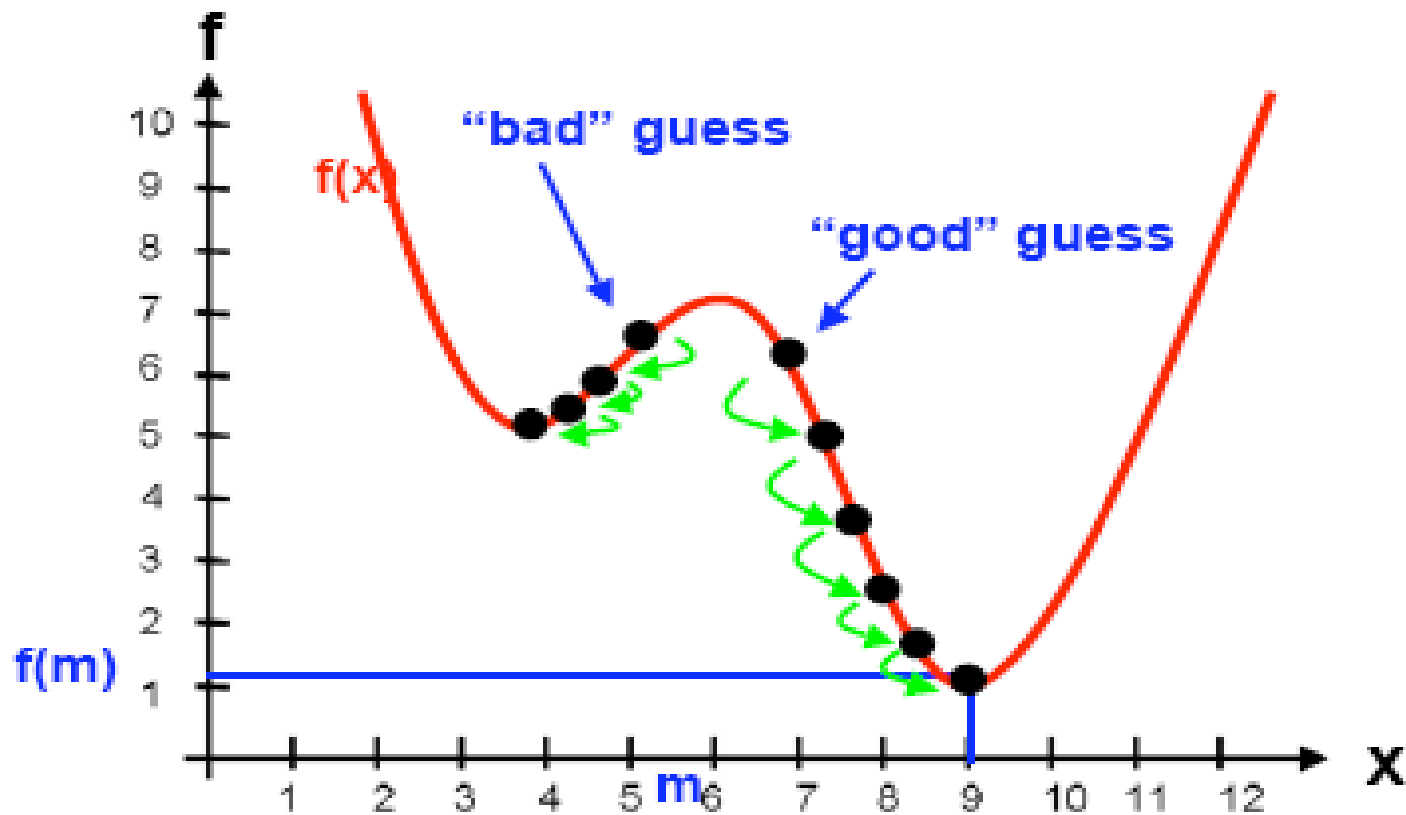
- If the feasible region is non-convex, then it is not possible to draw a line between any two possible solutions without that line leaving the set of possible solutions;
- If we start at point A, we could find a solution at point B;
- We cannot find a better solution from point B if we only look for changes which reduce the objective function;
- We cannot get to point C, the true optimum solution;



Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

- Unit commitment algorithms have variables controlling whether generators are on or off which can only take values 0 or 1;
- As there are no solutions allowed between 0 and 1, this is non-convex;
- This is because they violate the requirement that a point between 0 and 1 be in the feasible region;
- An **LP** could be solved for each permutation of '0,1' variables;
- But that goes back to brute force enumeration methods;
- Hence use of Lagrangian Relaxation and Mixed Integer Programming

A look at Lagrangian Relaxation



- Consider the objective function in the SEM
 - Schedule Generators, minimising MSP Production Costs
 - Subject to -
 - *Generation scheduled = MSP Schedule Demand;*
 - *Generators scheduled between min and max availability;*
 - *Generators output respects ramp rates;*
 - *Generators observe min up and min down times;*

- Consider the objective function in the SEM
 - Schedule Generators, minimising MSP Production Costs
 - Subject to -
 - *Generator energy limits no exceeded*
 - *Reservoir levels in Pumped Storage are observed*
 - *Interconnector ramp rates are followed*
 - Where MSP Production Costs = running costs and **Start** costs

- Development of LR program
 - Joseph Louis Lagrange;
 - Italian mathematician, worked in Berlin and France in the later 18th century;
 - Developed Lagrange Multipliers to assist in solving problems of variational calculus with integral constraints;
 - First used in Operations Research by Hugh Everett in 1959.
 - Application of Lagrange Multipliers to the objective function provided good sub-optimal solutions, in practical times;

Solver Choice in the SEM: LR vs. MIP

Pick Starting Point



Solver Choice in the SEM: LR vs. MIP

- Generator Units committed in priority order;
- Units ordered by average cost at max availability;
- Inter-temporal constraints ignored in this first pass;
- Including Min up, min down, start up costs, ramp rates, etc;
- Run an Economic Dispatch (**LP**) based on this commitment;
- Estimate Lagrange Multipliers based on this input;

Solver Choice in the SEM: LR vs. MIP

Pick Starting Point

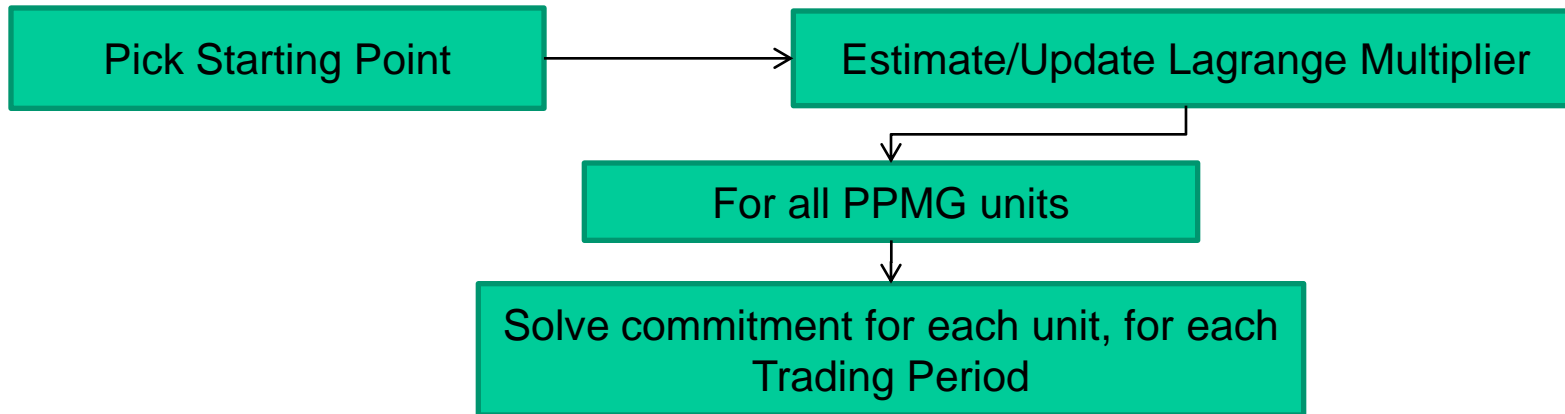


Estimate/Update Lagrange Multiplier



- System Lambda estimated;
- Estimate is smaller of
 - largest incremental cost of the units committed, and
 - incremental cost of generation of the first uncommitted unit, available in the priority list;
- System Lambda used as first Lagrange Multiplier in first **LR** iteration

Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

- The **LR** program decomposes the larger problem into smaller sub-problems;
- Optimises one unit at a time;
- Using coupling constraints to model the requirements of the larger problem;

- Problem 1 –
- Minimise $(\text{Cost} * \text{Output})_{\text{uh}1} + (\text{Cost} * \text{Output})_{\text{uh}2}$
- Subject to –
 - $(\text{Output}_{\text{uh}1} + \text{Output}_{\text{uh}2}) = \text{System Requirement};$
 - $\text{Output}_{\text{uh}1} \leq \text{Availability}_{\text{uh}1} * X;$
 - $\text{Output}_{\text{uh}2} \leq \text{Availability}_{\text{uh}2} * X;$
- Where X is commitment decision (0,1)

➤ Problem 2 –

➤ Minimise $(\text{Cost} * \text{Output})_{uh1} + (\text{Cost} * \text{Output})_{uh2}$
+ $LM_1(\text{System Load} - \text{Output}_{uh1} - \text{Output}_{uh2})$
+ $LM_2(\text{Availability}_{uh1} * X - \text{Output}_{uh1})$
+ $LM_3(\text{Availability}_{uh2} * X - \text{Output}_{uh2})$

➤ Where X is commitment decision (0,1)

➤ And **LM** is Lagrange Multiplier

➤ Problem 3 –

➤ Minimise LM_1 (System Load)

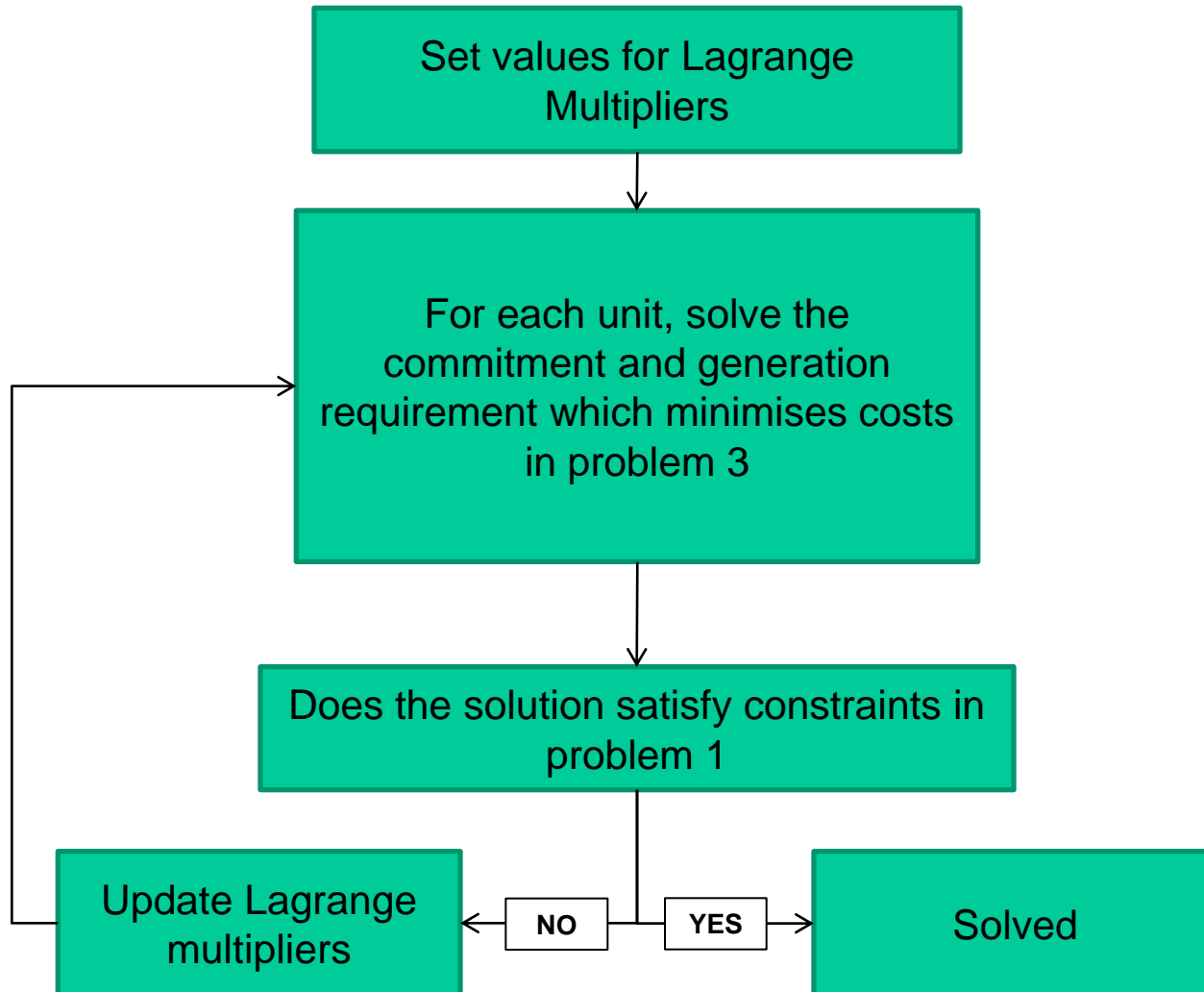
$$+ (\text{Cost} * \text{Output})_{uh1} - LM_1(\text{Output}_{uh1}) + LM_2(\text{Availability}_{uh1} * X - \text{Output}_{uh1})$$

$$+ (\text{Cost} * \text{Output})_{uh2} - LM_1(\text{Output}_{uh2}) + LM_3(\text{Availability}_{uh2} * X - \text{Output}_{uh2})$$

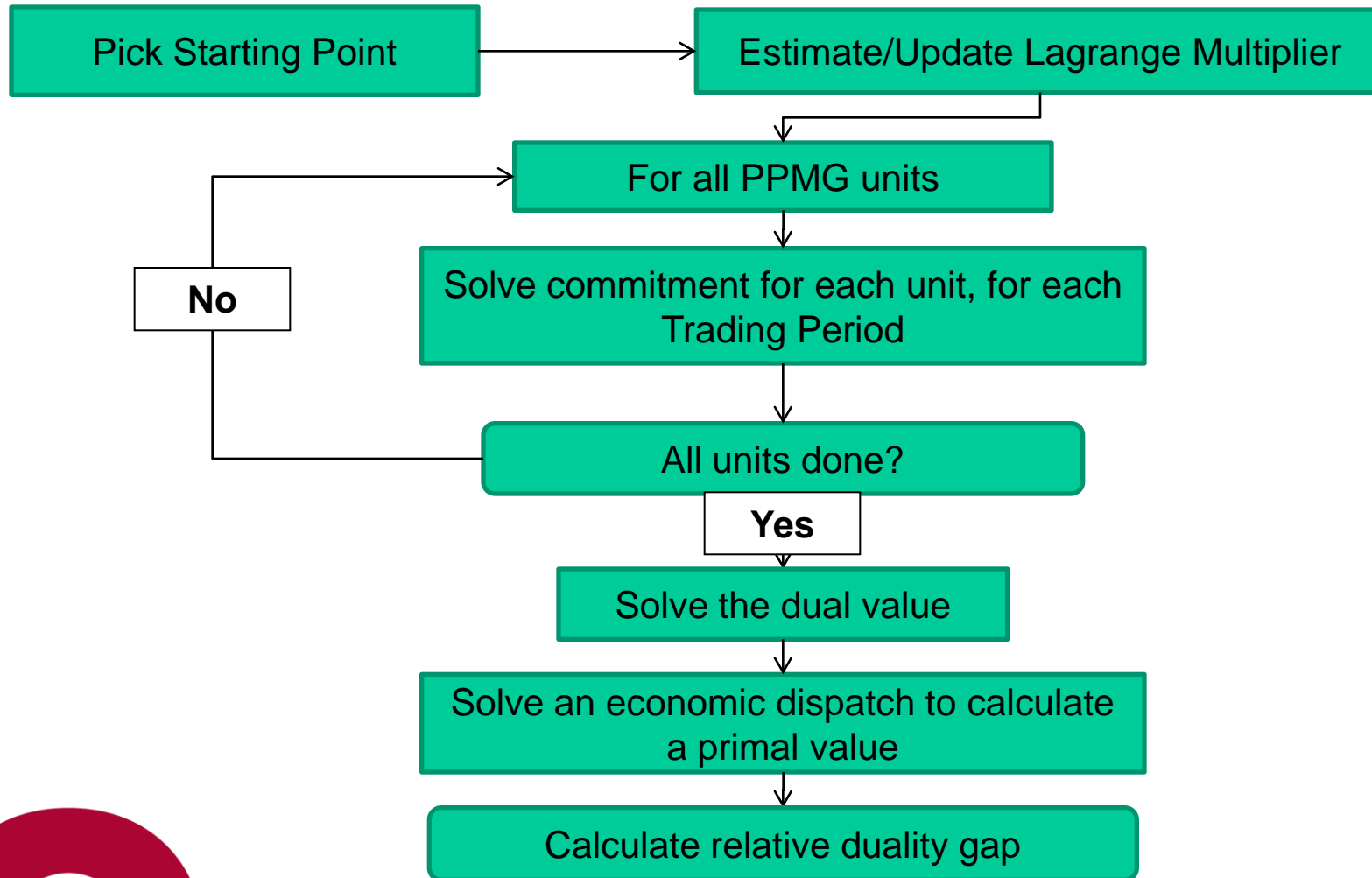
➤ Where X is commitment decision (0,1)

➤ And LM is Lagrange Multiplier

Solver Choice in the SEM: LR vs. MIP

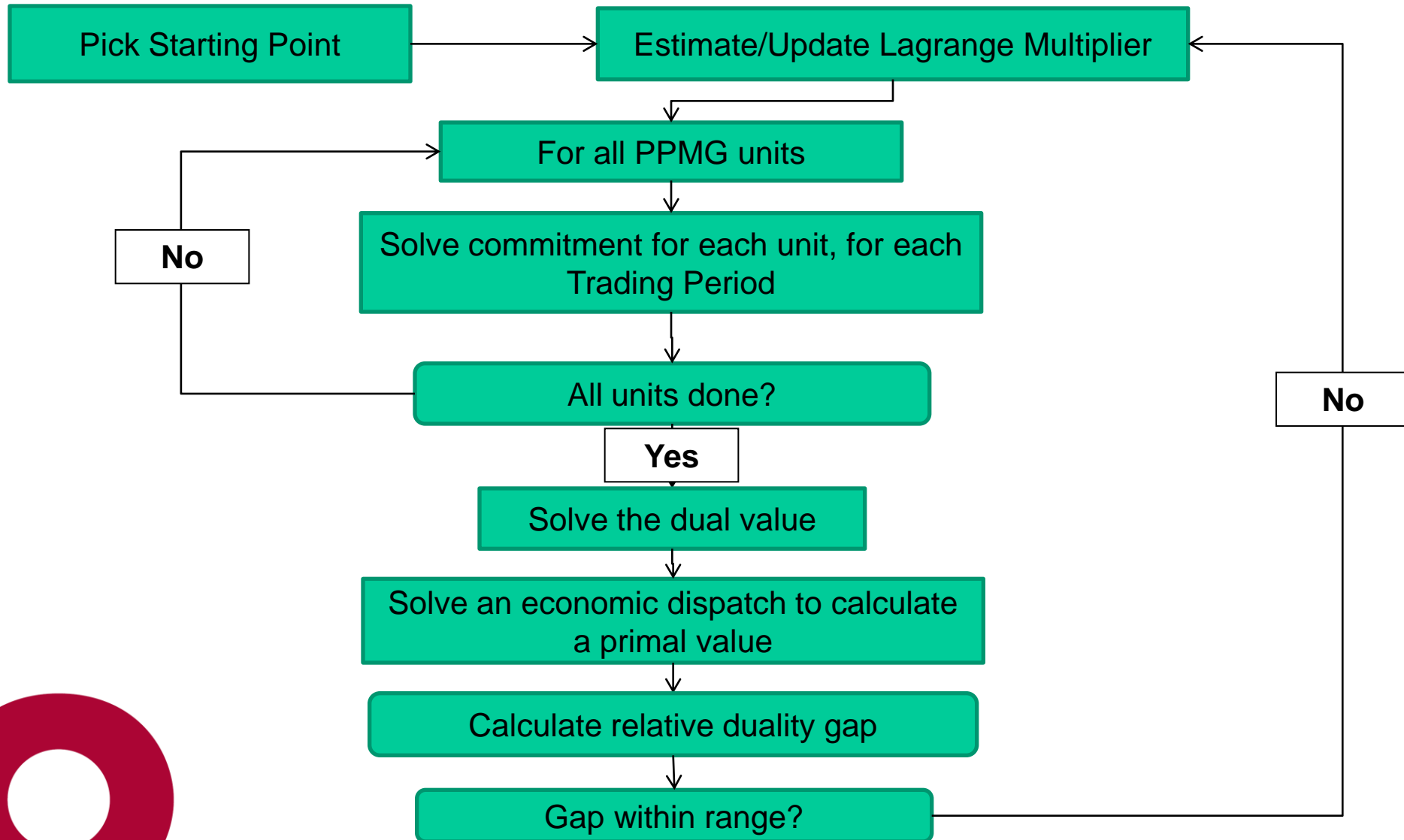


Solver Choice in the SEM: LR vs. MIP



- Primal dual gap sometimes used – relative difference between values of primal problem and dual problem;
- SEM implementation has additional stage to ensure feasible solution from **LR** process;
- Convergence is measured by the stability of the solution;
- Solution converges if
 1. it is feasible, and
 2. small changes to the **LMs** aimed at improving the objective function will make the solution become infeasible;

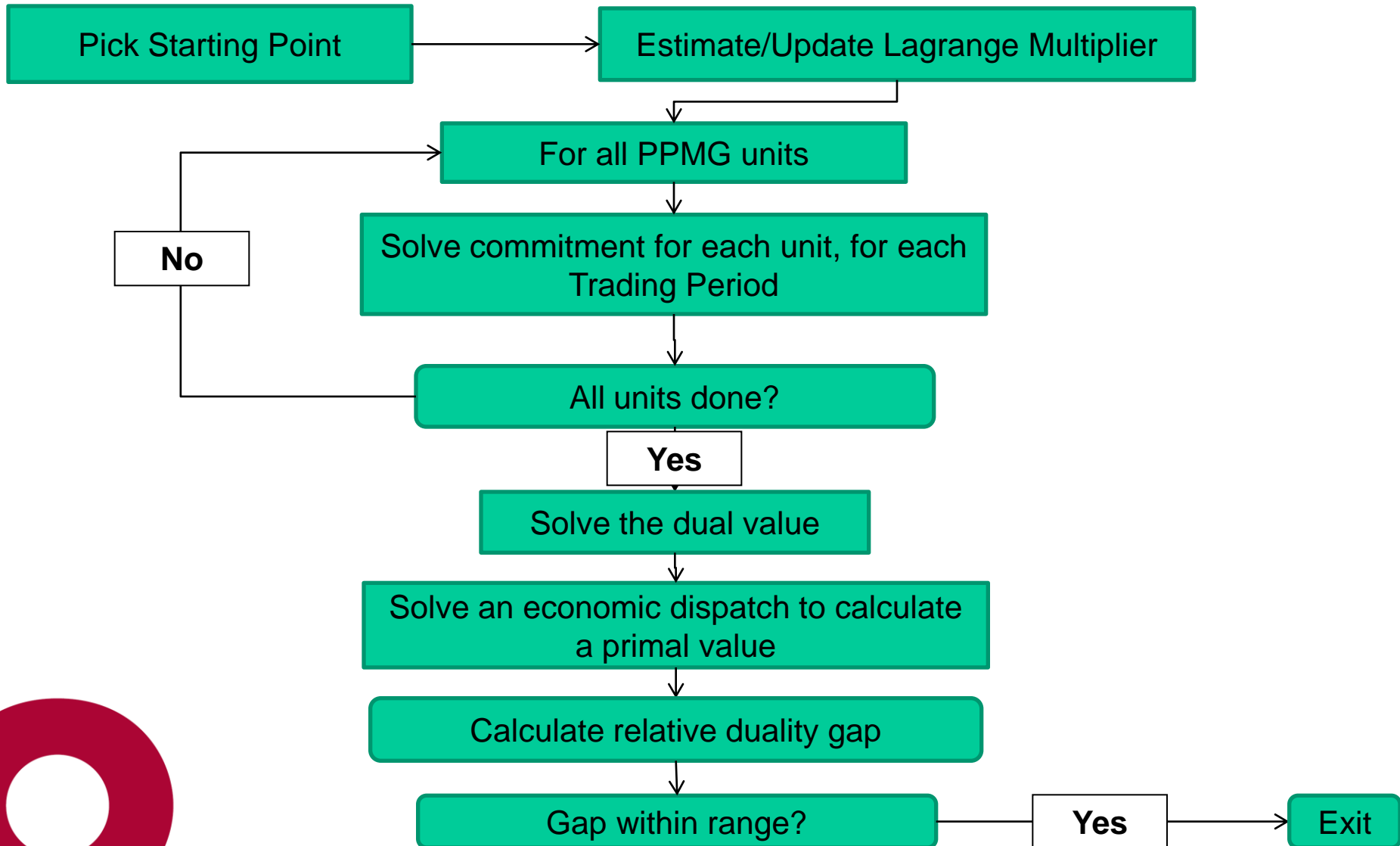
Solver Choice in the SEM: LR vs. MIP



Solver Choice in the SEM: LR vs. MIP

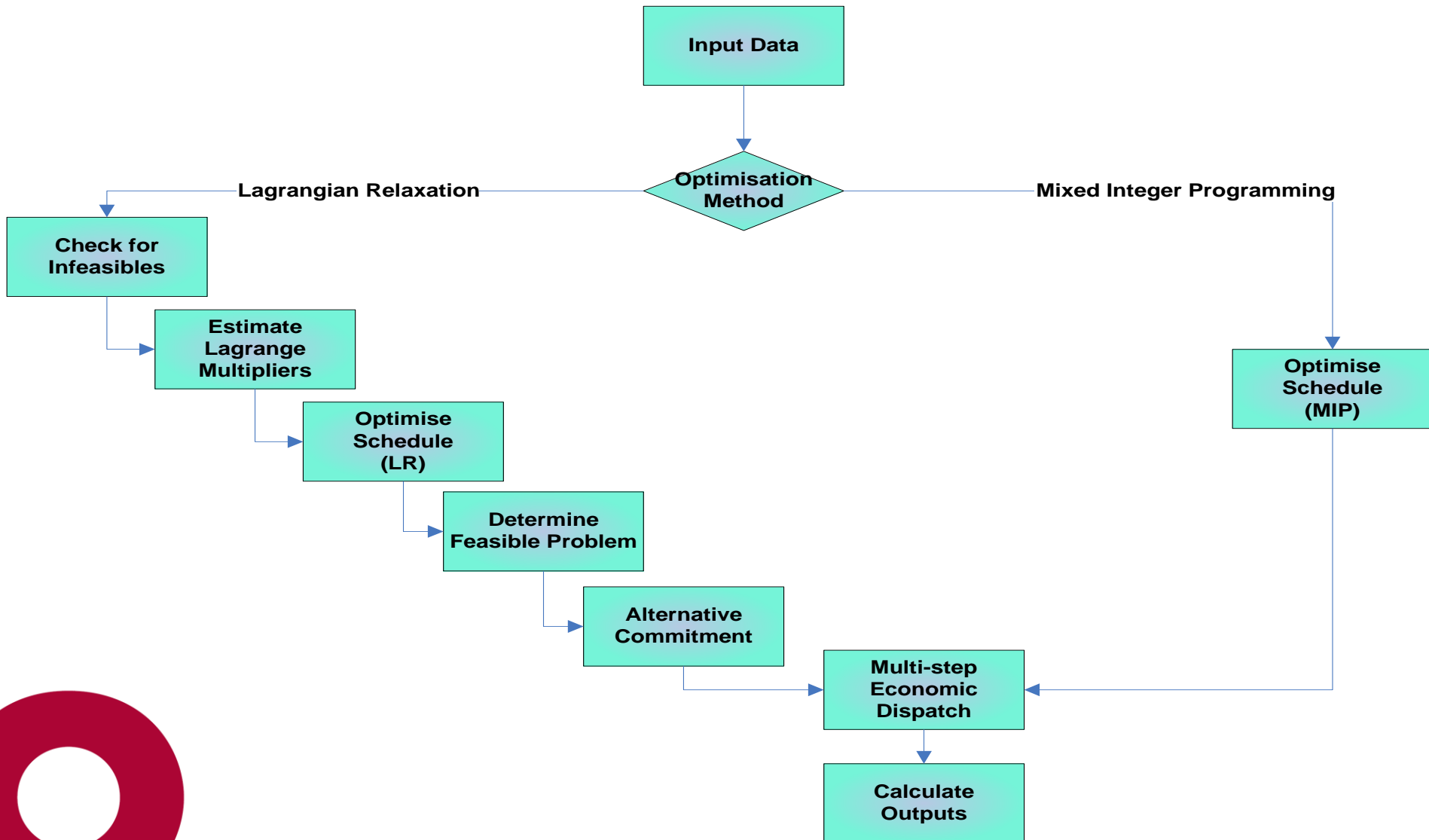
- Update **LM** values;
- Uses sub gradient method;
- Change to the **LM** is proportional to difference between available amount and required amount for the constraint;
- **LM** will increase if contribution of a unit to a constraint is less than the requirement;
- **LM** will decrease if contribution of a unit to a constraint is greater than the requirement;

Solver Choice in the SEM: LR vs. MIP



- Optimality of final solution?
- Solution is suboptimal *but* feasible;
- Further heuristics applied to this result;
- Alternative Commitment or ALTCOM phase of program;

Solver Choice in the SEM: LR vs. MIP



- What is ALTCOM?
- Single Unit Dynamic Programming;
- Starts with the schedule produced by the LR procedure;
- Considers whether small changes in the schedule will improve the overall schedule;
- Since the logic will implement a change in the schedule only if it results in reduced cost, the changes can only result in improvements;

- What is ALTCOM?
- Checks units that cycle on and off;
- Checks units that are brought on for short runs – peakers;
- Checks where unit brought on while another unit is brought off;
- Examines unit by unit, trading period by trading period;
- Units that have inter-temporal constraints cannot be changed – Energy Limited, Pumped Storage;

- Lagrangian Relaxation, in summary:
 - Optimises one unit at a time;
 - Uses coupling constraints to connect single unit optimisation to objective function;
 - Final LR outcome further enhanced by heuristics in ALTCOM phase;
 - Should find feasible *but* suboptimal solution;

A look at Mixed Integer Programming



Solver Choice in the SEM: LR vs. MIP

- Mixed Integer Programming , emerged in the mid 1950s;
- An extension of Linear Programming;
- Two main algorithms developed -
 - “Branch and bound”, and
 - Cutting planes;
- Solutions were combined to form the “Branch and Cut” algorithm;



Solver Choice in the SEM: LR vs. MIP

- In this, the program discards large sections of the search space by comparing upper and lower bounds;
- Allows the program to search more solutions than an **LR** program;
- More likely to reach global optimality;
- But, until recently, not practical for market operations;



Solver Choice in the SEM: LR vs. MIP

- Table here shows improvements in MIP solution times with a given problem over a period of years;

Version	Time (seconds)
CPLEX 1.0 (1988)	57840
CPLEX 3.0 (1994)	4555
CPLEX 5.0 (1996)	3835
CPLEX 6.5 (1999)	165

- Improvements driven through enhancements to the algorithm but also computer technology (CPU power and memory);

Source – MIP: Theory and Practice – Closing the Gap, Bixby, Fenelon, Gu, Rothberg & Wunderling

➤ Bullets below show further improvements in CPLEX solution times since 2000;

- CPLEX 12.2 (2010): 50% overall, 2.7X on 1,000 seconds and up
- CPLEX 12.0 (2009): 30% overall, 2X on 1,000 seconds and up
- CPLEX 11 (2007): 15% under one minute, 3X on 1-60 minutes, 10X on one hour and up
- CPLEX 10 (2006): 35% overall, 70% on “particularly difficult models”
- CPLEX 9 (2003): 50% on “difficult customer models”
- CPLEX 8 (2002): 40% overall, 70% on “difficult problems”
- CPLEX 7 (2000): 60% on “hard mixed integer problems”

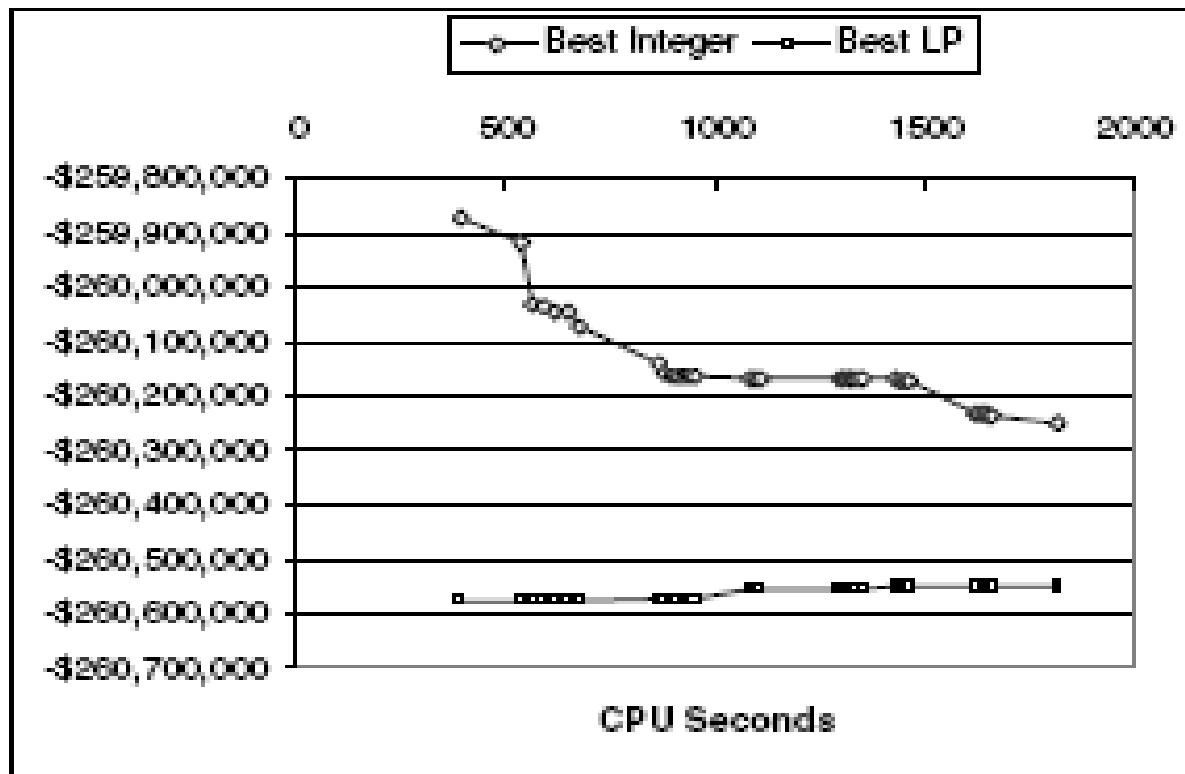
Source – <http://www-01.ibm.com/software/integration/optimization/cplex-optimization-studio/cplex-optimizer/cplex-performance/>

Solver Choice in the SEM: LR vs. MIP

- Takes a starting point with a relaxed version of the problem as a benchmark;
- Not perfect solution but an approximation of it;
- All integer variables allowed to have continuous values;
- Establishes lower bound on cost of objective function;
- Then, uses Branch & Bound/Branch & Cut to search out best solution;
- Solutions are measured against a “MIP Gap”
- The MIP Gap = $\text{ABS}[(\text{Best Integer}-\text{Best LP})/\text{Best LP}] * 100 (\%)$ (where Best Integer represents solution found by Branch & Cut, and Best LP represents relaxed version)

Solver Choice in the SEM: LR vs. MIP

➤ Comparison of Best Integer to Best LP

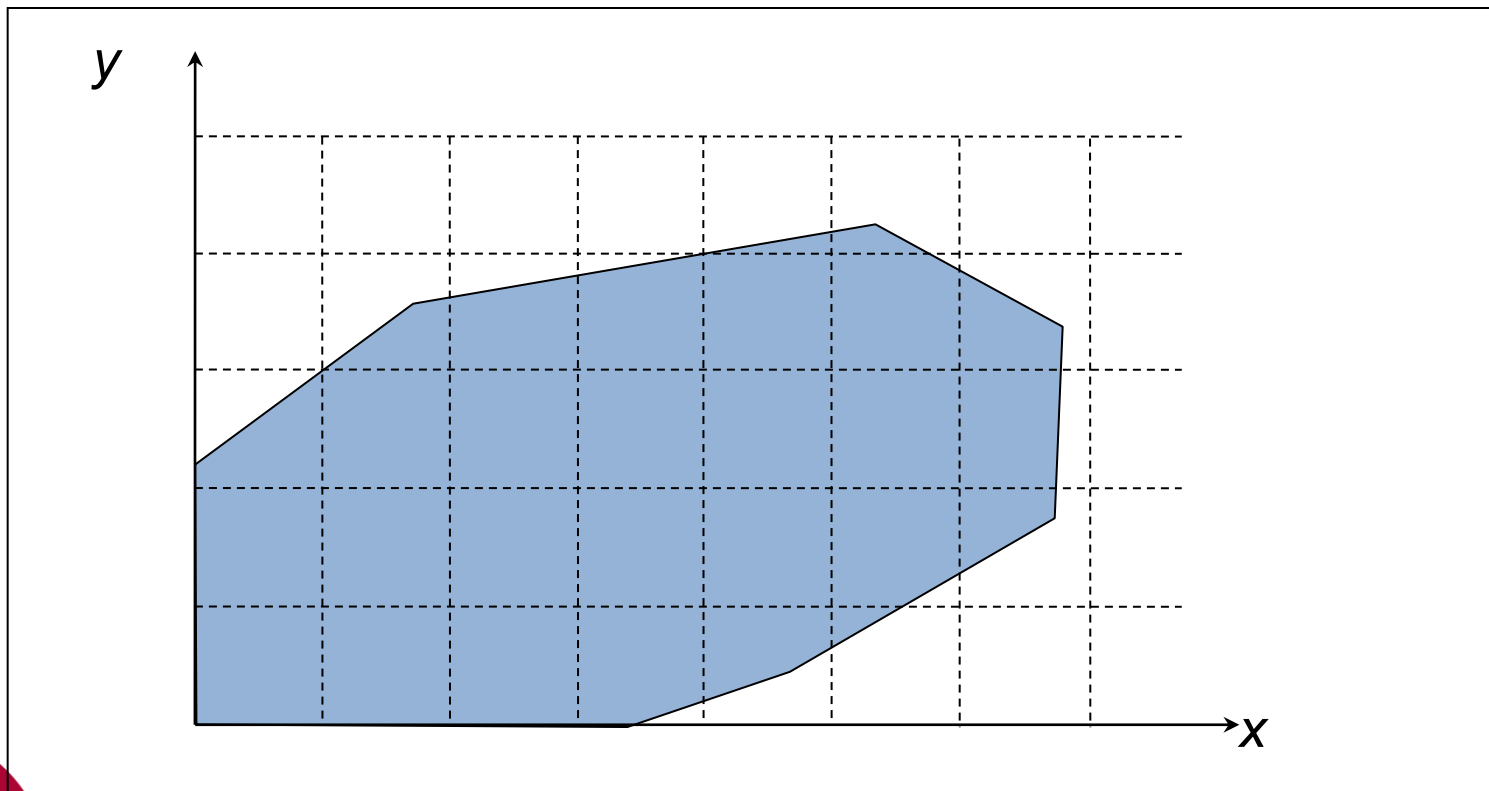


Source – [A Mixed Integer Programming Solution for Market Clearing and Reliability Analysis - Streiffert, Philbrick, & Ott](#)

- Possible solutions are in blue shaded region and at an intersection point of the grid;



➤ Branch and Cut -

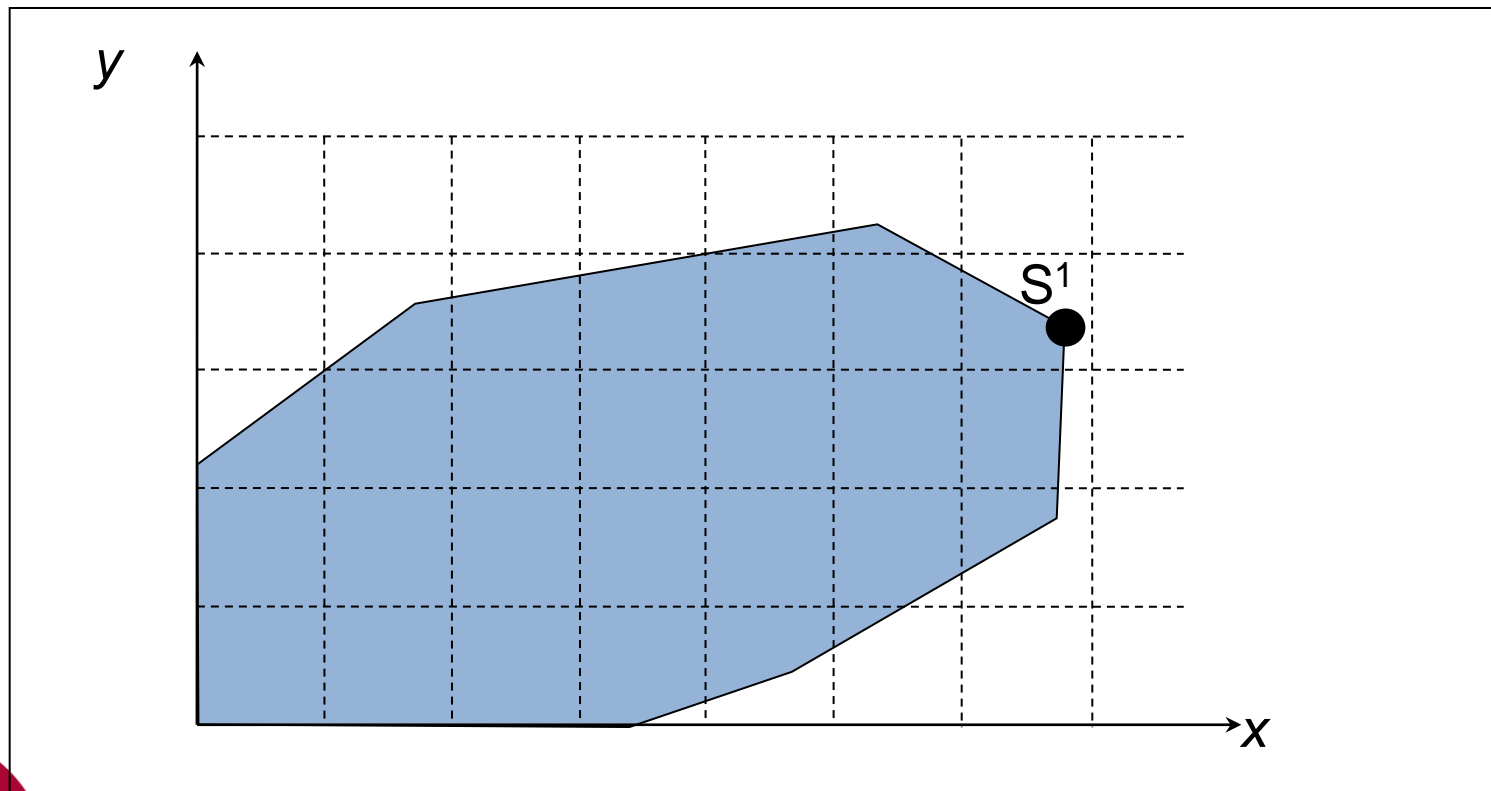


Solver Choice in the SEM: LR vs. MIP

- Possible solutions are in blue shaded region and at an intersection point of the grid;
- S1 is the solution to the “relaxed” linear programming (LP) problem;
- But not an integer solution;



➤ Branch and Cut -

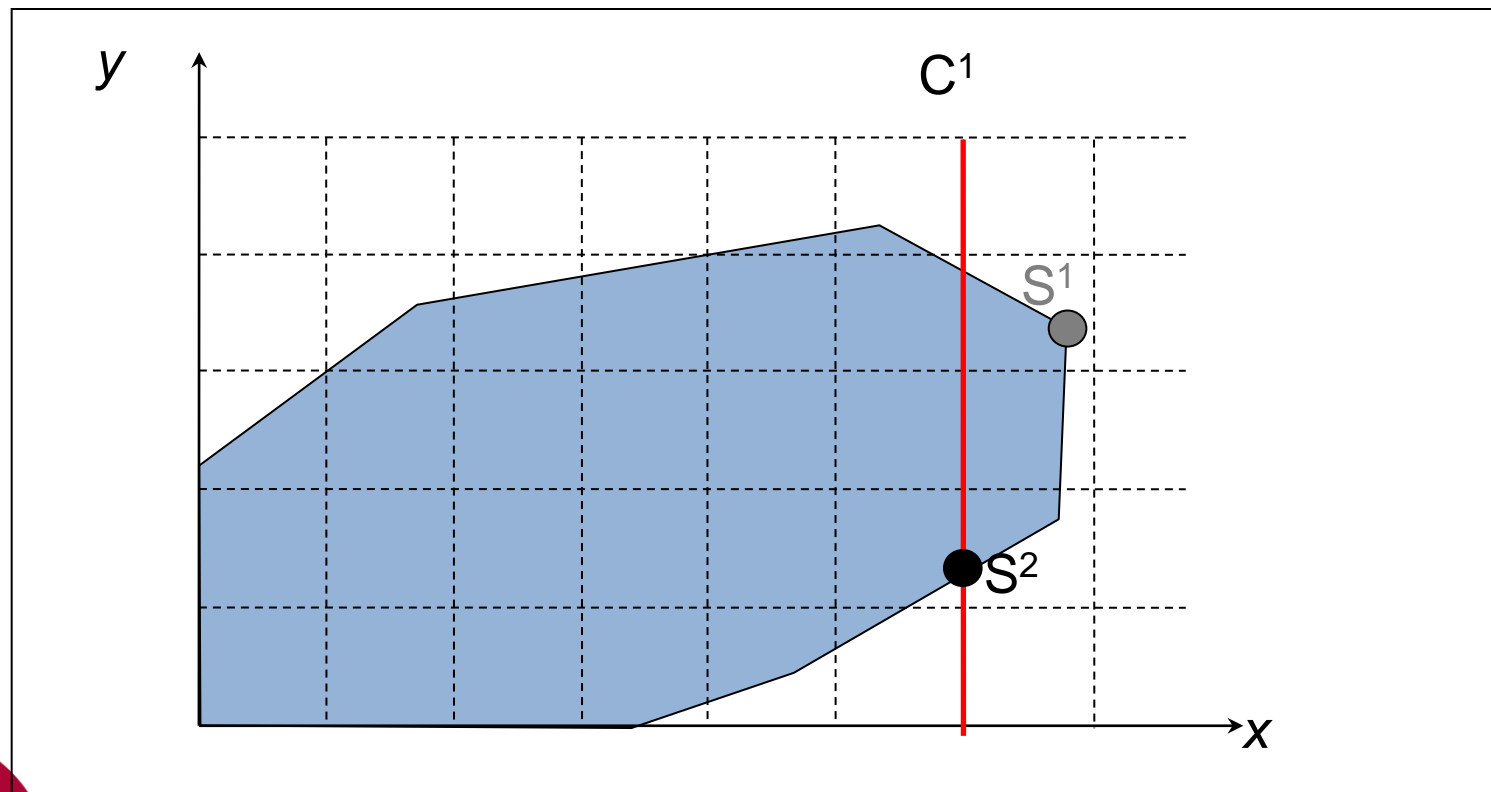


Solver Choice in the SEM: LR vs. MIP

- Possible solutions are in blue shaded region and at an intersection point of the grid;
- S1 is the solution to the “relaxed” linear programming (LP) problem;
- But not an integer solution;
- Constraint C1 eliminates some potential solutions but eliminates no integer solutions.
- The new LP solution is at S2; however, still not an integer solution;



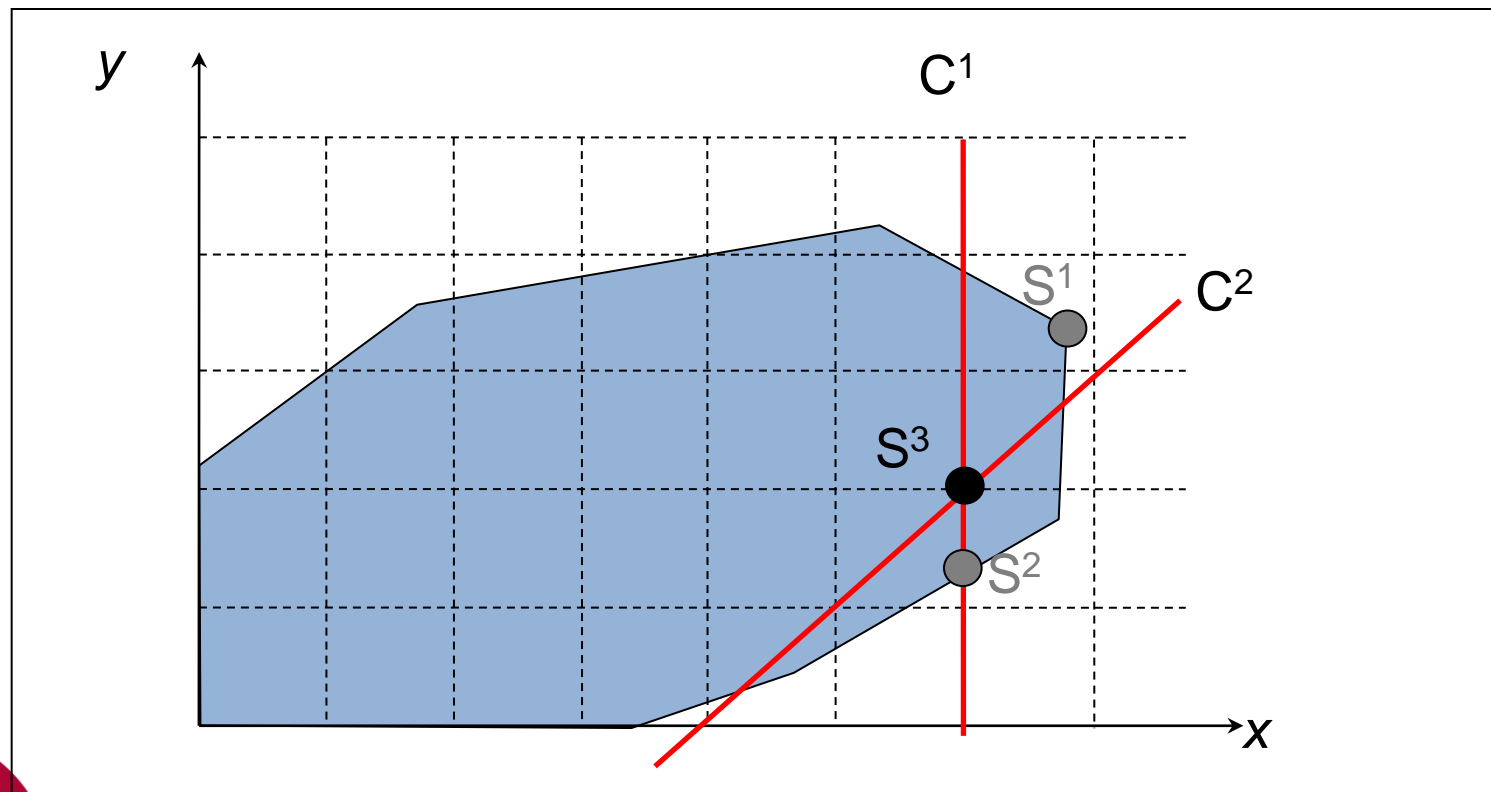
➤ Branch and Cut -



Solver Choice in the SEM: LR vs. MIP

- Possible solutions are in blue shaded region and at an intersection point of the grid;
- S1 is the solution to the “relaxed” linear programming (LP) problem;
- But not an integer solution;
- Constraint C1 eliminates some potential solutions but eliminates no integer solutions.
- The new LP solution is at S2; however, still not an integer solution;
- Constraint C2 eliminates more potential solutions but eliminates no integer solutions. The new LP solution is at S3;

➤ Branch and Cut -

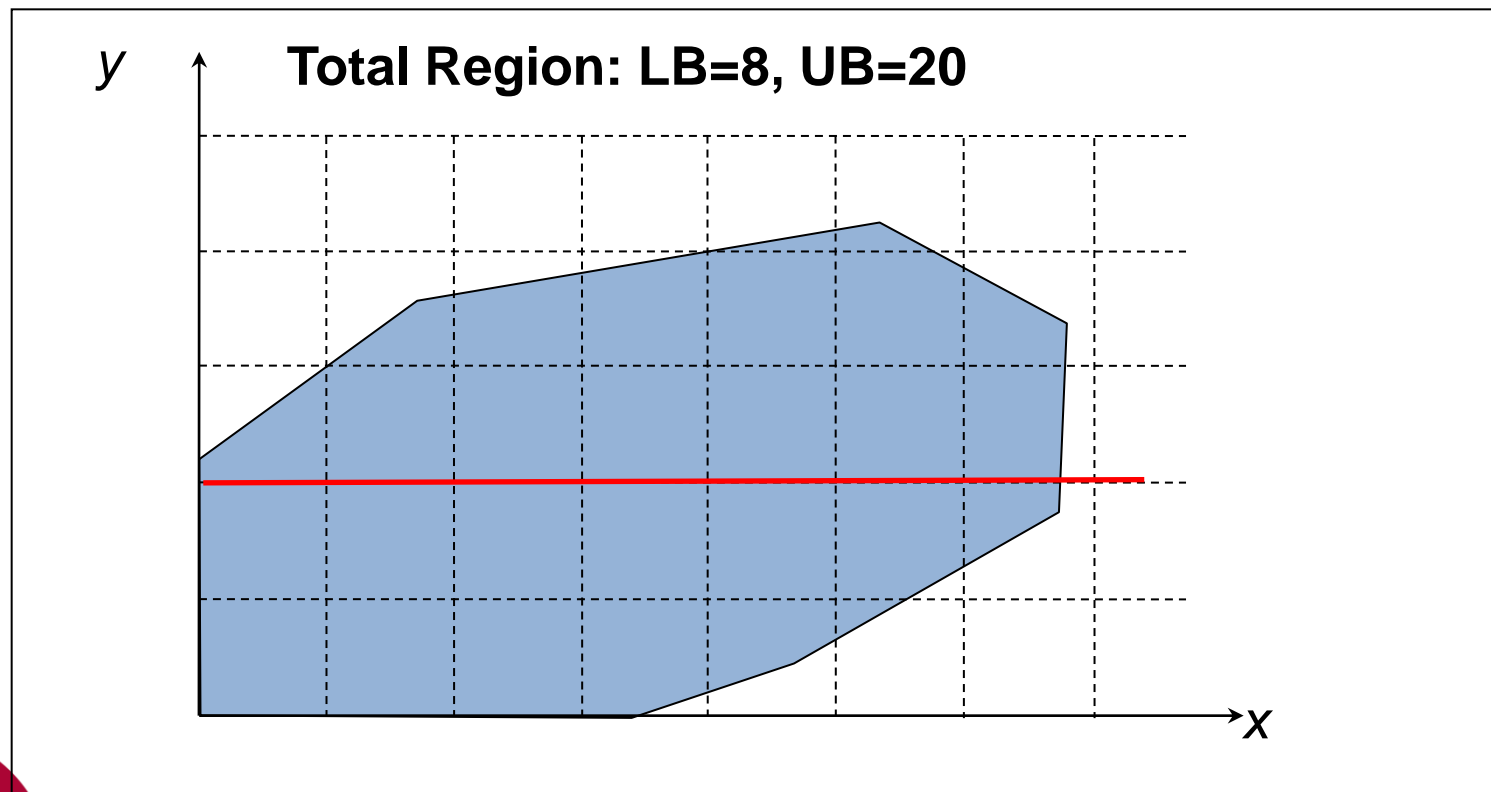


Solver Choice in the SEM: LR vs. MIP

- Branch and bound is used when branch and cut is exhausted.
- The region is divided into two;



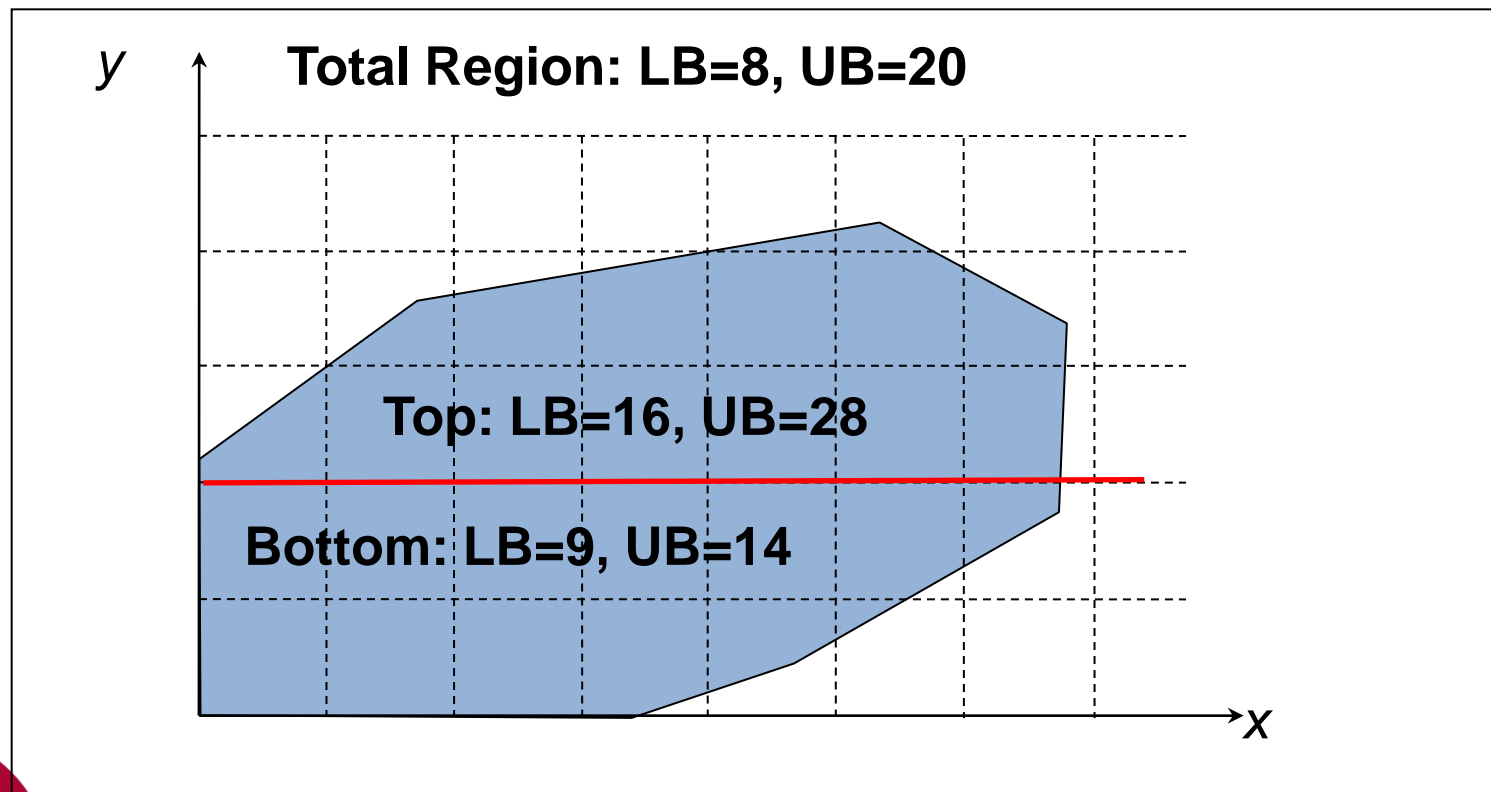
➤ Branch and bound -



Solver Choice in the SEM: LR vs. MIP

- Branch and bound is used when branch and cut is exhausted.
- The region is divided into two;
- A theoretical upper and lower bound of the solution cost is determined for each region.

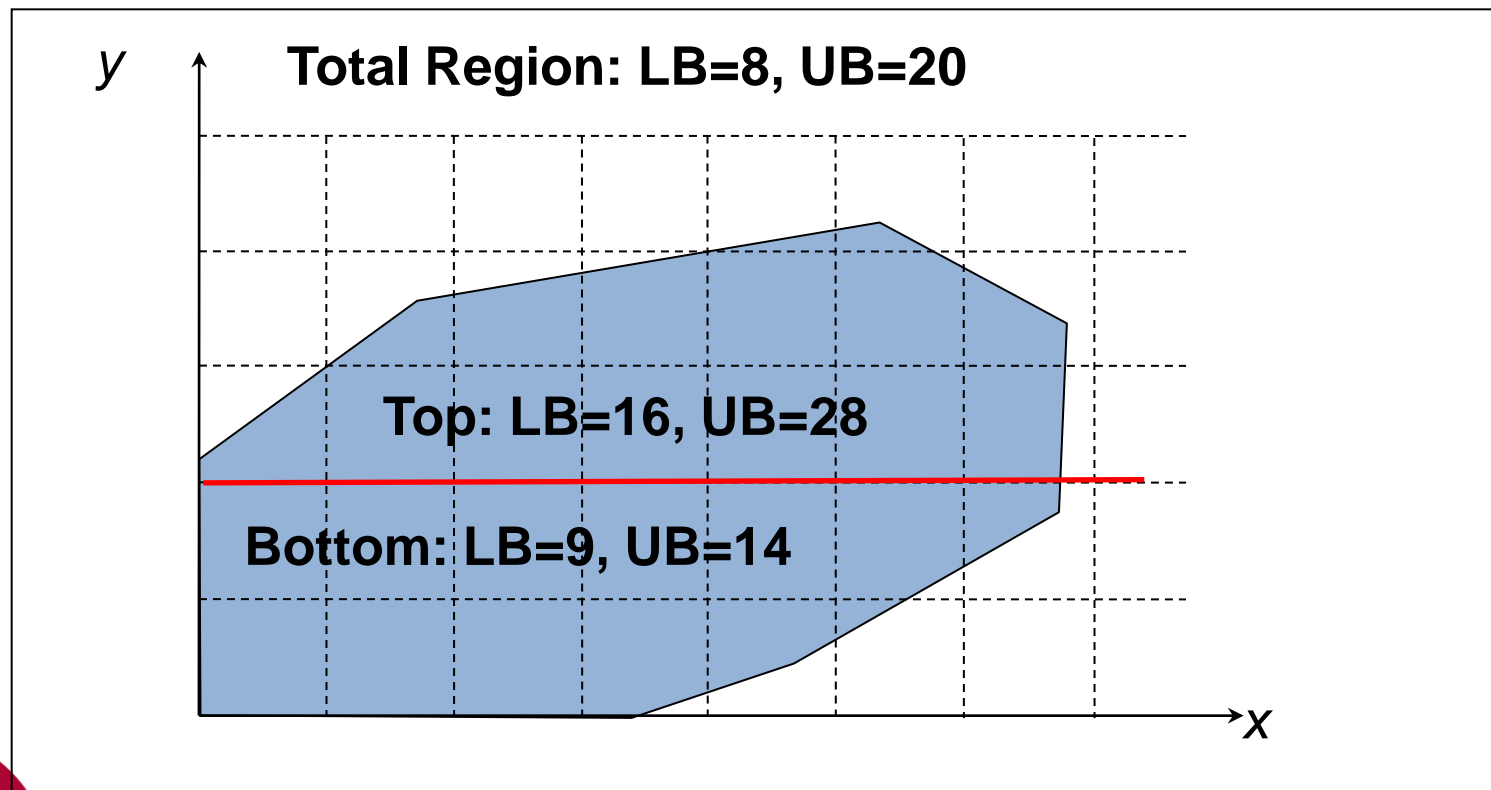
➤ Branch and bound -



Solver Choice in the SEM: LR vs. MIP

- Branch and bound is used when branch and cut is exhausted.
- The region is divided into two;
- A theoretical upper and lower bound of the solution cost is determined for each region.
- For the case shown, the optimum solution must lie in the lower region as the upper bound on cost is less than the lower bound of the top region.
- Branch and cut can be used again now.

➤ Branch and bound -



Solver Choice in the SEM: LR vs. MIP

- As noted - **more likely** to reach global optimality;
- However;
- Solution needs to run until MIP Gap = 0;
- This is still impractical for market operations;
- Market Operators (incl. SEMO) use *timeouts* and *Optimality Gap* settings;
- SEMO optimality gap = **1%**
- Standard timeout = **300 seconds**
- Once processing is terminated, solution will **more likely** be suboptimal

Summing up



➤ LR

- Does not solve the true problem.
- Hence, will not find true optimum solution.
- But can find a very good solution fast.



➤ MIP

- Always reduces the region over which it searches so that the true optimum solution remains in that region.
- Hence, MIP **can** find the true optimum solution.
- But can be very slow.
- Time limits and optimality gaps mean it may not find the optimum solution.

What?

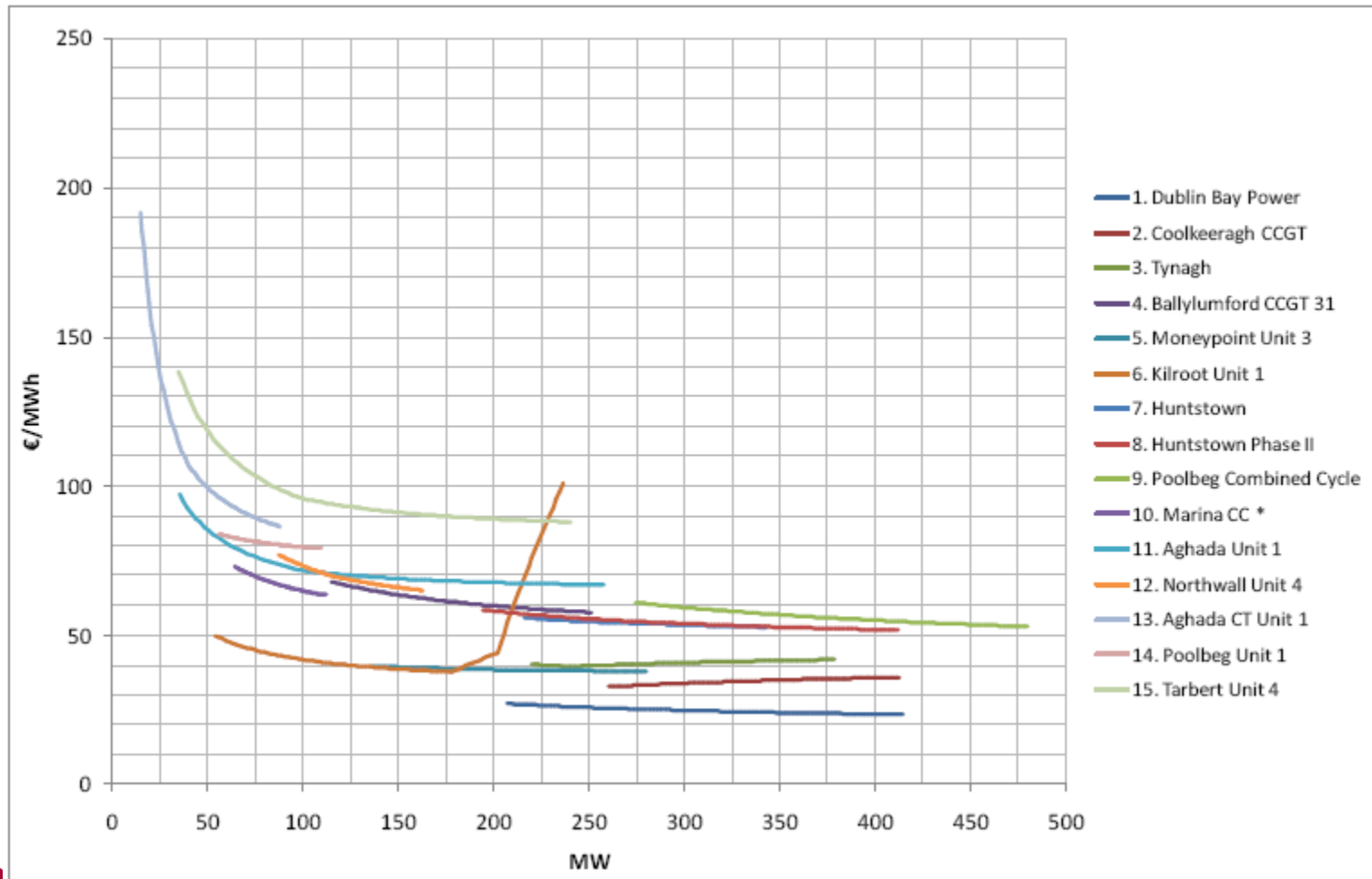


- What do other markets do?
 - For centrally committed energy markets, LR is still widely in use;
 - PJM moved to a MIP solver in 2005;
 - California ISO moved to a MIP solver in 2009;
 - New York ISO using LR but considering move to MIP;
 - Non-centrally committed markets can use LP solutions (as not commitment decisions required);

- Why did we pick LR?
 - SEM is a centrally committed market design;
 - During the design phase (2005 to 2007), PJM had only just moved to MIP;
 - It was considered that the SEM would not be a guinea pig for a MIP solution;
 - LR selected as principal “solver of choice” because of stability of software;
 - ABB software can use both a bespoke LR or CPLEX’s MIP solver;

- The SEM experience with solvers
 - Very early in market operations, we noted bidding patterns that created more difficult problems for the solver;
 - Kilroot bidding with respect to their output on Oil vs. output on Gas added complexities;

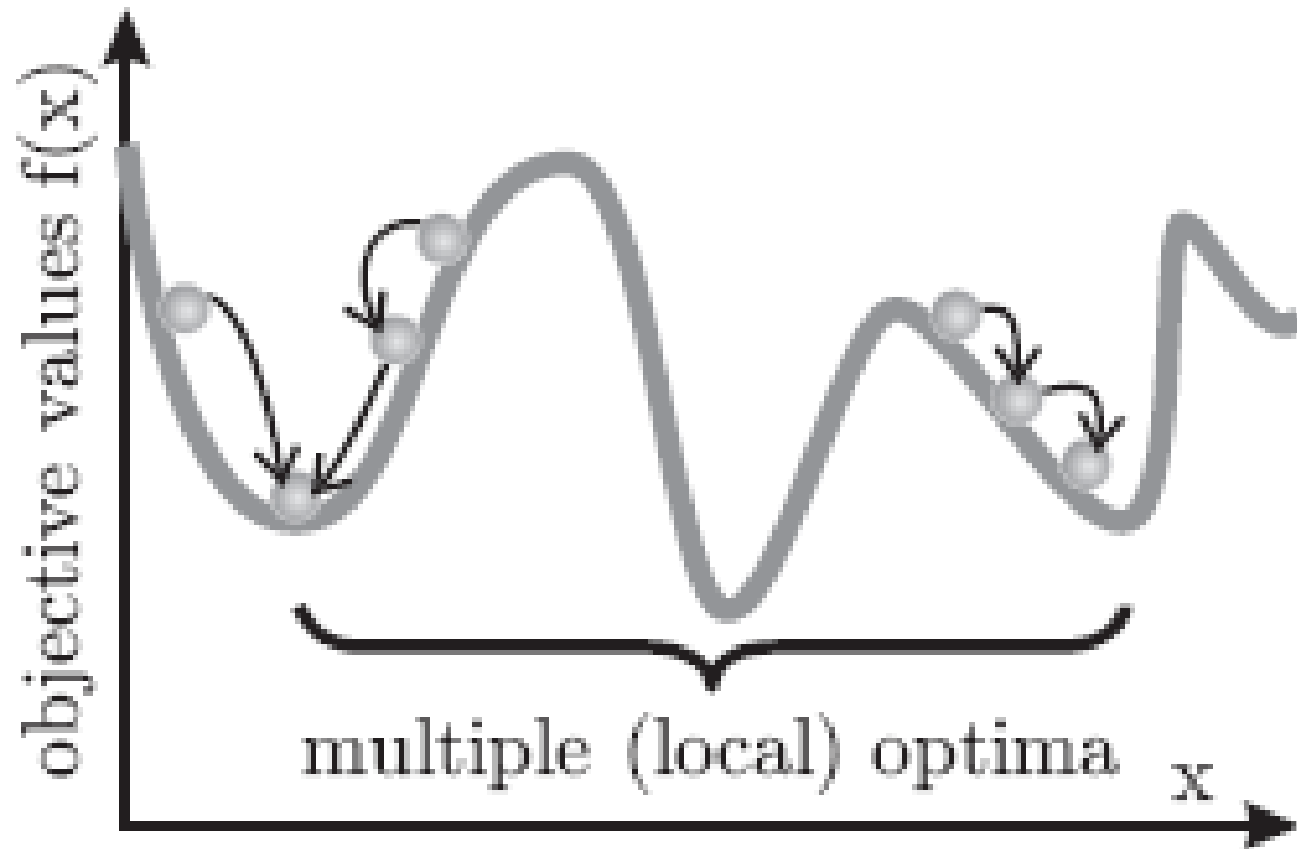
Solver Choice in the SEM: LR vs. MIP



Generator Commercial Offer Data – MMU Report

- The SEM experience with solvers
 - Very early in market operations, we noted bidding patterns that created more difficult problems for the solver;
 - Kilroot bidding with respect to their output on Oil vs. output on Gas added complexities;
 - While normal solutions are “multi-modal”; that is, there can be many similar local optimal solutions -

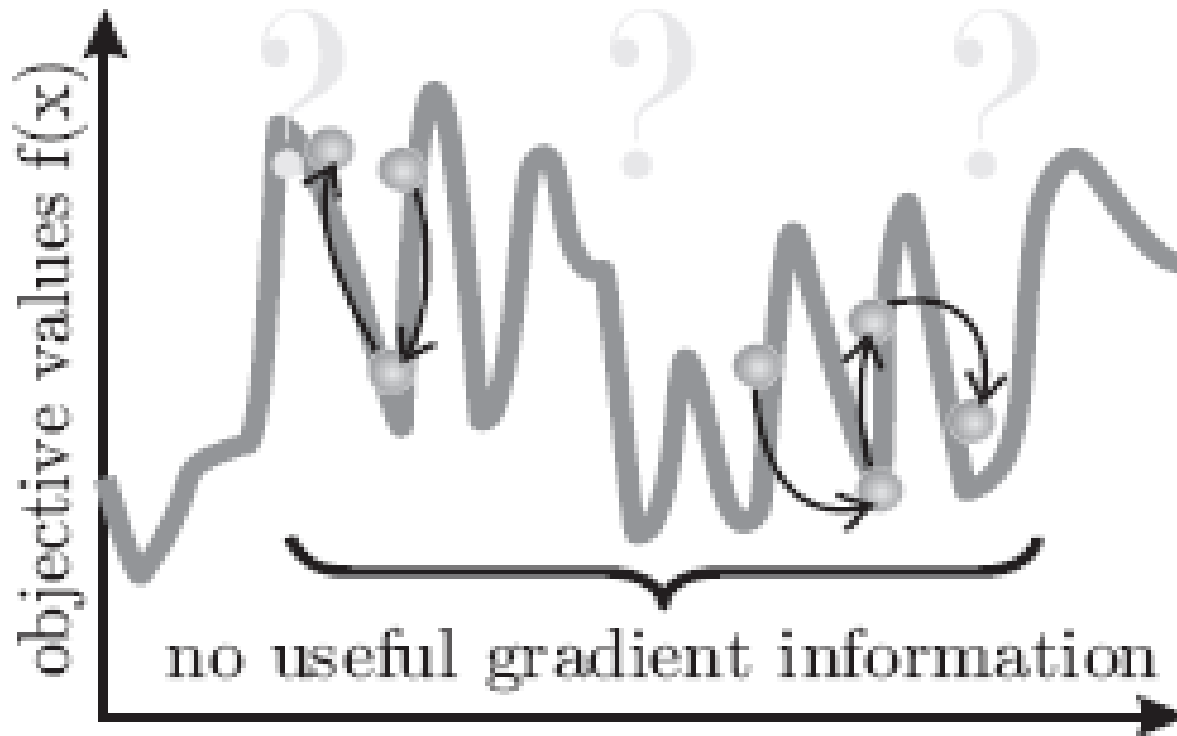
Solver Choice in the SEM: LR vs. MIP



Source – *Why is Optimization Difficult?* Weise, Zapf, Chiong & Nebro

- The SEM experience with solvers
 - Very early in market operations, we noted bidding patterns that created more difficult problems for the solver;
 - Kilroot bidding with respect to their output on Oil vs. output on Gas added complexities;
 - While normal solutions are “multi-modal”; that is, there can be many similar local optimal solutions –
 - This bidding caused more “rugged” solutions; that is, the gradients between solutions points become steeper;

Solver Choice in the SEM: LR vs. MIP



Source – *Why is Optimization Difficult?* Weise, Zapf, Chiong & Nebro

- The SEM experience with solvers
 - Early experience showed LR frequently converging prematurely on sub-optimal solutions;
 - Experience of Price Spikes ($> \text{€}500$) in the SEM;
 - This led to the development of the SEM policy on use of MIP;
 - MIP would be used when certain price events observed for study purposes;
 - If assessment of output showed that T&SC obligations better met by MIP solver, then this would be published.

- Why do the LR vs. MIP study?
 - SEMO committed to industry that we would undertake a comprehensive review of the solvers;
 - Study run between Q3 2009 and Q1 2010;
 - Scope of study presented to Participants in 2009;
 - Schedule outputs analysed in detail across areas defined in scope;

Study Scope

A total of **170** dates (154 Ex-Post Initial and 16 Ex-Ante) were chosen from the start of the Market up to and including **August 2009**.

The choice was based on the following criteria:

- Seasonal/Holiday Periods – Summer, Winter, Easter, Christmas;
- Weekdays, Weekends;
- High/Low Wind days, increased wind penetration;
- High/Low Activity on Interconnector;
- Unexpected treatment of Energy Limited Plant, Pumped Storage;
- Price events – spikes, high uplift, zero shadow price, etc

Study Scope

The following runs were completed for all dates:

- **LR** (re-run as the published one may have been completed with a different version of the software)
- **MIP300** seconds
- **MIP600** seconds (run only if MIP300 did not reach the optimality gap required)
- **MIP1800** (run only in case MIP600 did not reach the optimality gap required)

Also for limited number of dates:

- Altering LR ALTCOM parameters,
- Running consecutive Trading Days in blocks,
- Altering MIP Optimality Gap, and
- Amending commercial offer data for Hydro generators

Study Environment

- All runs were carried out on the Market Clearing Engine Certification (MCEC)
- A dedicated study environment for the duration of the study
- All study cases days were completed using the latest certified version (1.4.11 – **7th July 2009**) of the MSP software. The MIP software used was CPLEX 10.0.
- It included a copy of the published runs for all identified study dates.

Comparative Analysis

Analysis was carried out on the following key areas:

- MSP Production Costs;
- System Marginal Prices;
- Revenue (Generator Revenue and Consumer Costs);
- Scheduling of Energy Limited Generators (Hydro Stations);
- Unit Commitment relative to fuel and station technology type;
- Constraint Payments;
- Timeout settings of the MIP Solver;
- Internal MSP software parameters

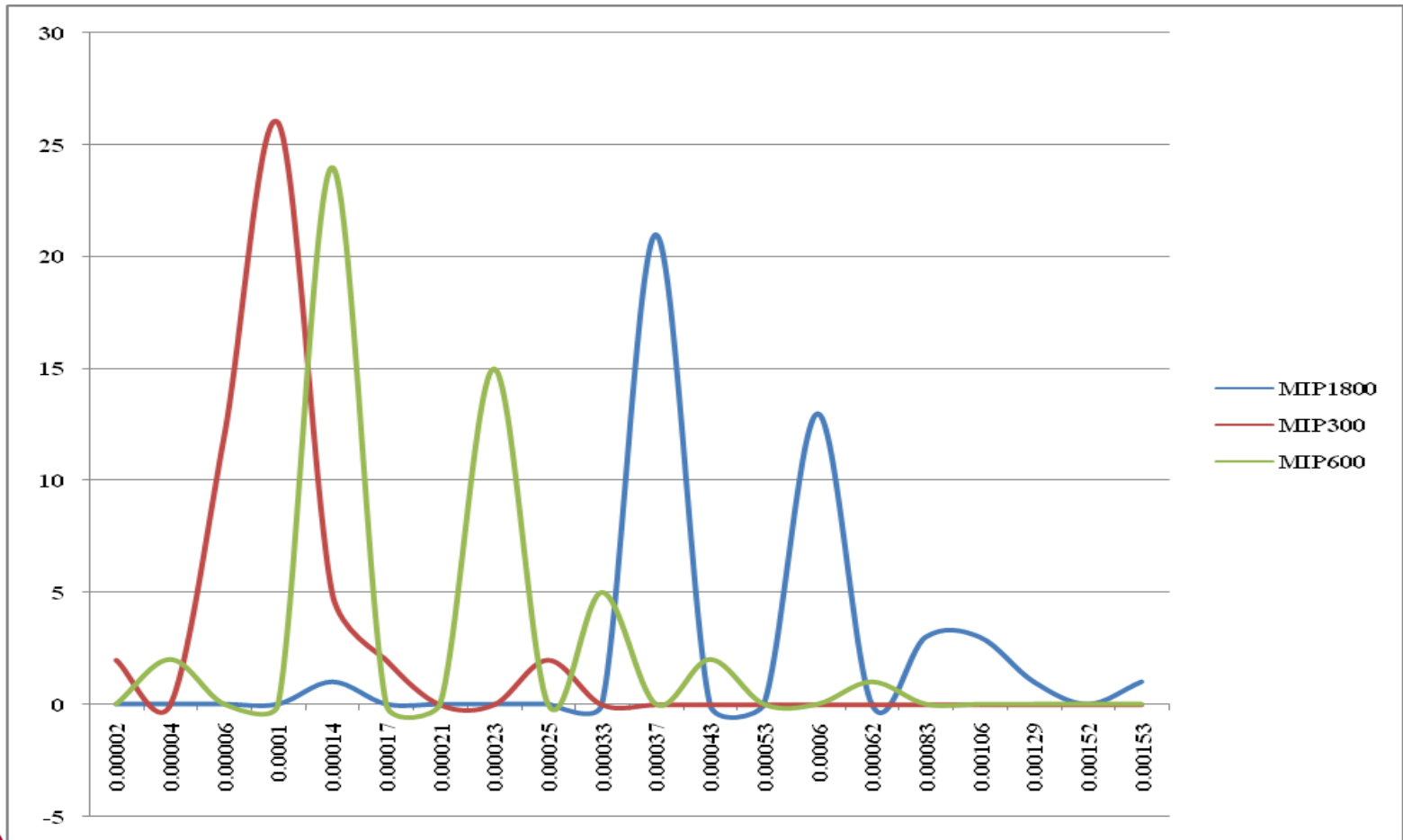
The economic downturn was taken into account throughout the analysis phase. The split was as follows; Pre (Dec 2007 to Jan 2009) and Post (Feb 2009 to Aug 2009)

MIP Timeout Settings Comparison

- MIP uses timeout settings and a convergence tolerance for Optimality Gap;
- This means the MIP solver will always terminate after one of the above parameters have been achieved before reaching a global optimal solution.
- Of the 154 study cases over 100 solved to within the convergence tolerance within the five minute setting.
- Of the remaining third of cases, 24 never solved to within the convergence tolerance even after thirty minutes.

<i>Timeout Settings</i>	<i>Study Runs completed</i>	<i>Solved to Optimality Gap</i>	<i>Success Rate</i>
MIP300	154	105	68.18%
MIP600	49	6	12.24%
MIP1800	42	18	42.86%

Solution Values



MSP Production Cost Comparison

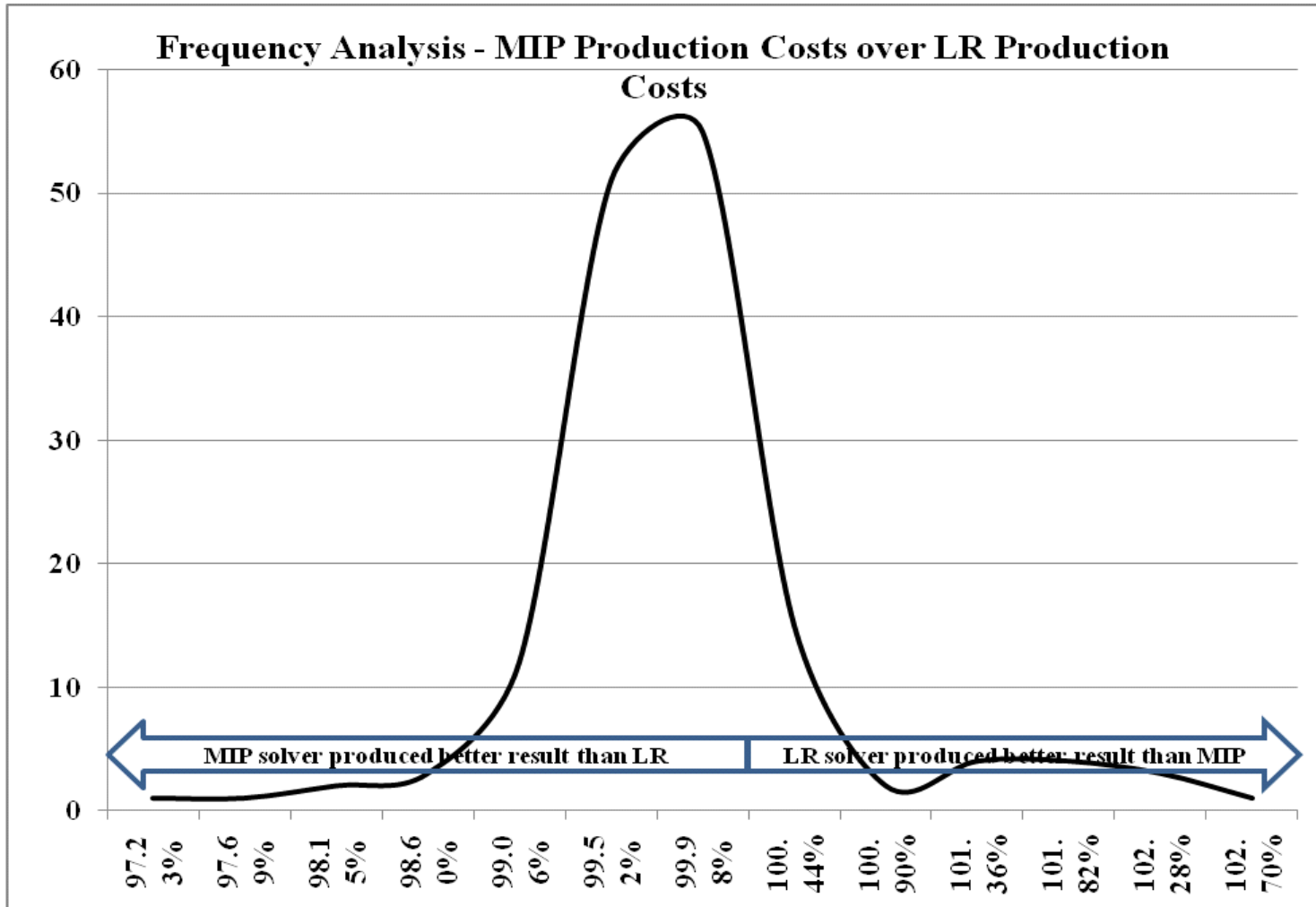
Section 4.67 in the Trading & Settlement Code dictates that :

'The high level objective of each run of the MSP Software when producing a Unit Commitment Schedule or Market Schedule Quantities,[...] is to minimise the aggregate sum of MSP Production Costs for all Price Maker Generator Units over a given Optimisation Time Horizon, subject to [...] constraints '

The following observations were made:

- In over 83% of study cases, there was an improvement in MSP Production Costs with the MIP solver.
- LR Production Costs is generally within 1% of MIP
- Small improvements in the MSP Production Cost can lead to large changes in the overall SEM outcomes with significant changes to Consumer Costs being observed.

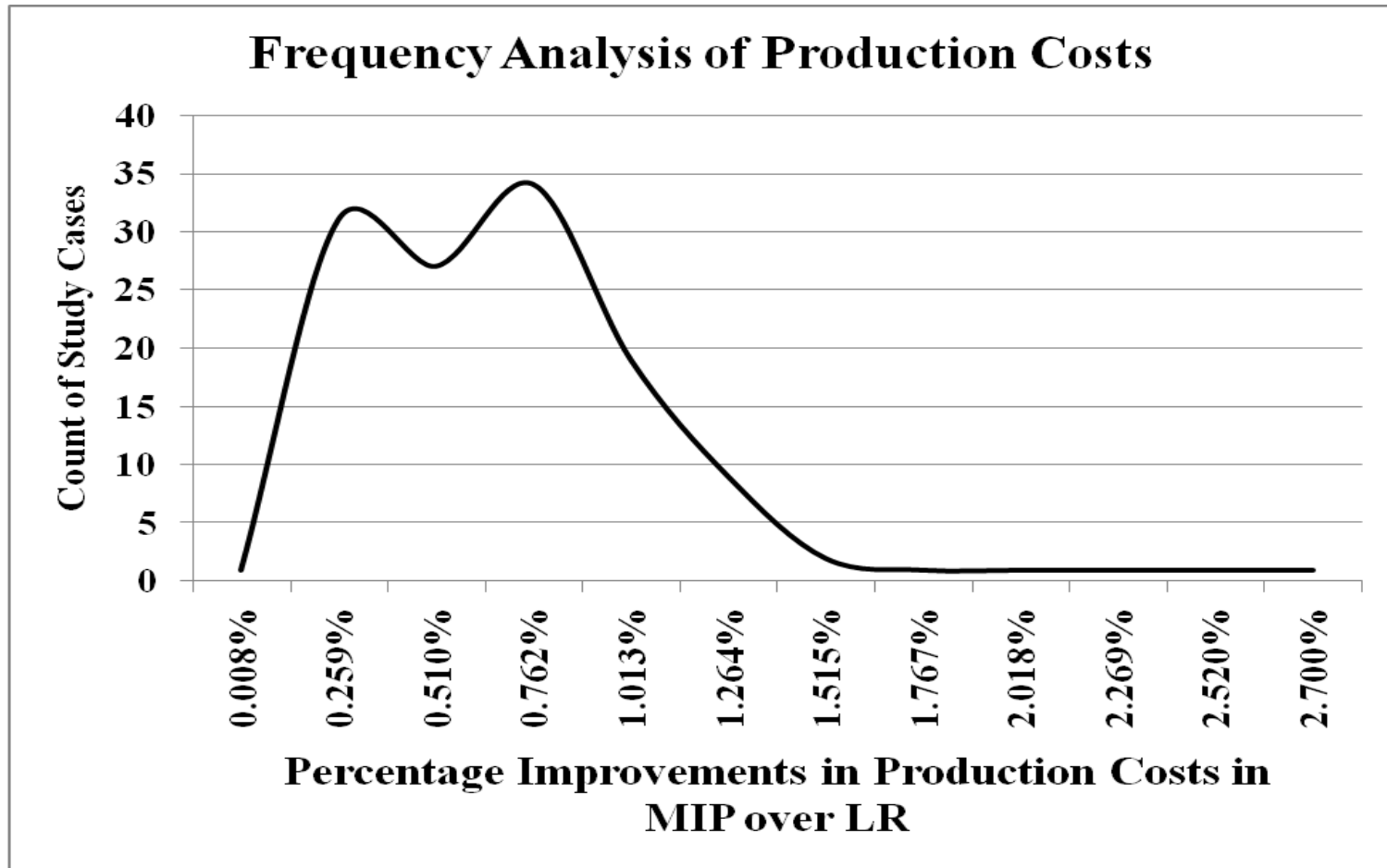
Solver Choice in the SEM: LR vs. MIP



MSP Production Cost Comparison

- The scale of the improvement can be quite small with most cases falling between 0 and 1% with a substantial number of cases (58) between 0 and 0.5%.
- The average improvement across all cases where MIP performed better was 0.591%.
- In monetary terms the daily average improvement was noted as €35,356.

Solver Choice in the SEM: LR vs. MIP



This graph represents the 83.117% of study cases which demonstrated an improvement.

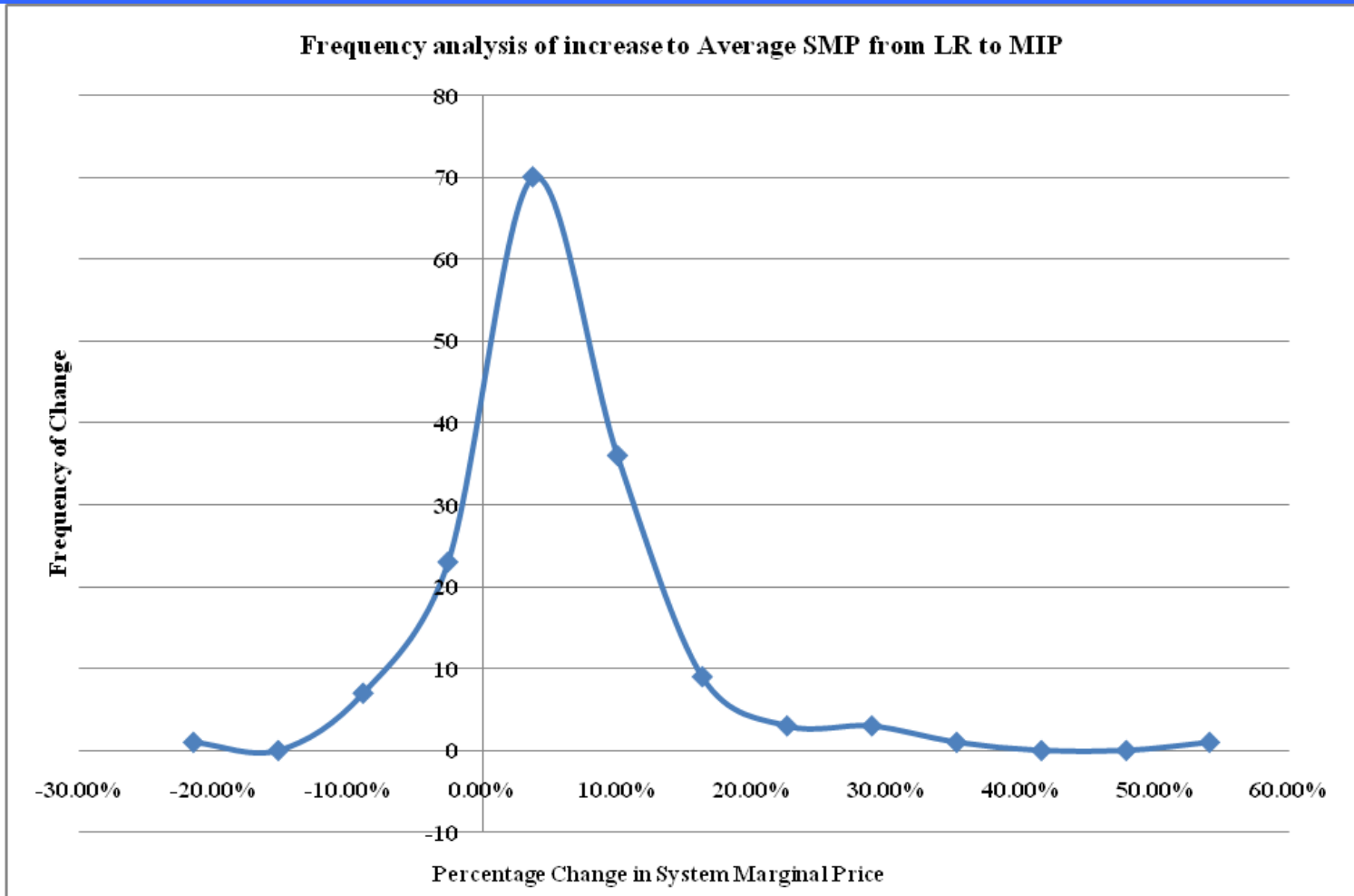
MSP Production Cost Comparison

- Average improvement on MSP Production Costs observed in solutions with the MIP solver over the LR runs was follows:
 - Pre Economic Downturn by € 38,097
 - Post Economic Downturn by € 24,603.18
- The maximum observed improvement in MSP Production Costs is €194,478.75 with the minimum being €456.52.

System Marginal Price Comparison

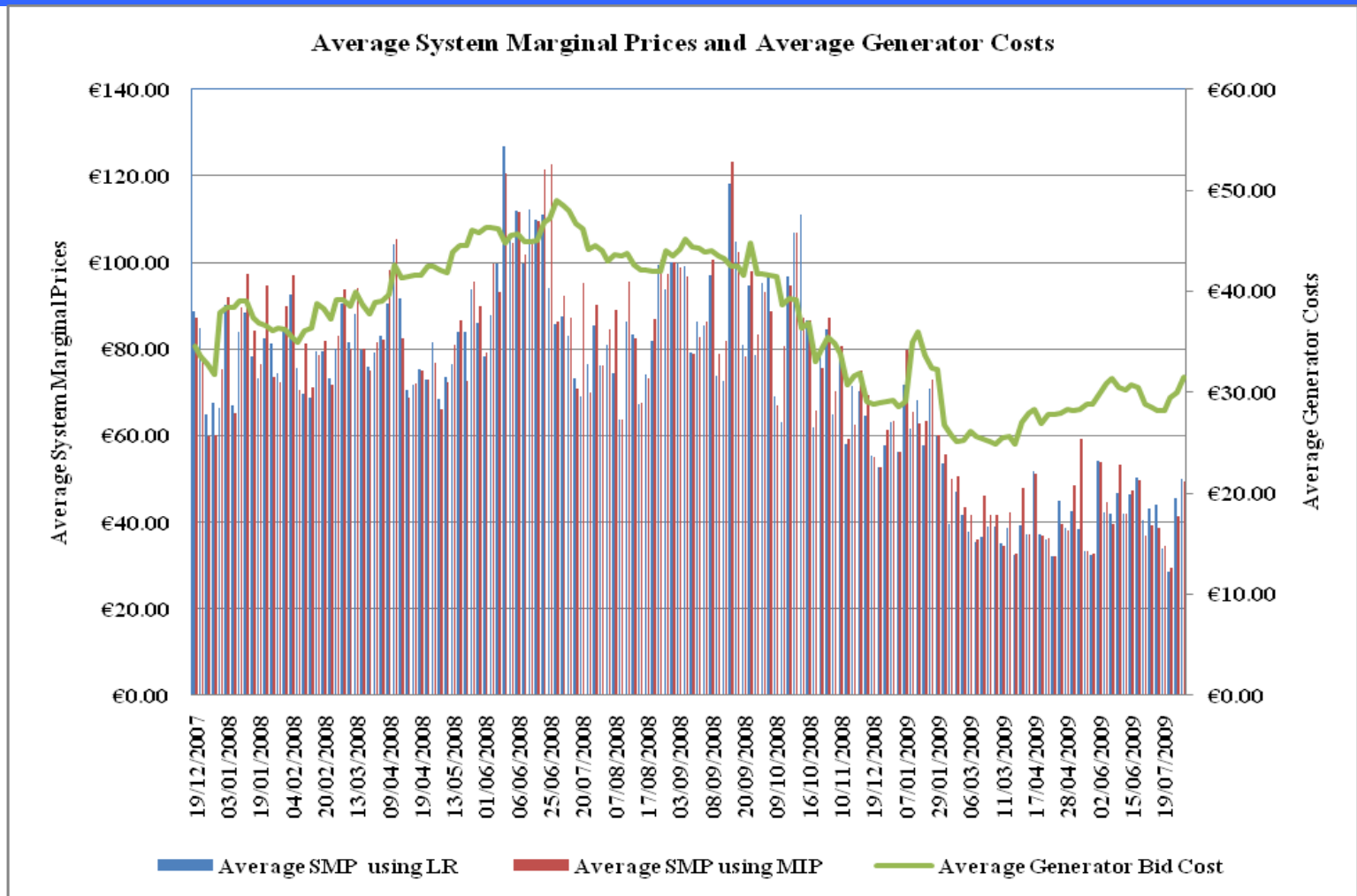
- The Average Daily System Marginal Price (SMP) increased in 57% of cases with the MIP solver over LR; however, the Maximum SMP decreased in 52% of the total number of cases with MIP.
- This shows a tendency of LR solver towards producing a lower average SMP but conversely more instances of higher Prices.
- Peak Prices are classed as SMP greater than €500. Peak Prices were observed in the LR and MIP runs as follows:
 - Pre Economic Downturn, there were 9 Peak Prices (5 in LR & 4 in MIP).
 - Post Economic Downturn, there were no Peak Prices.
- The Average Daily Uplift calculated is greater in 66.94% of LR cases. This percentage is based on the 121 cases when Uplift was calculated in both the LR and MIP runs.

Solver Choice in the SEM: LR vs. MIP



53% of cases show an increase in the daily average SMP of between 0% and 20% when using the MIP solver.

Solver Choice in the SEM: LR vs. MIP



Daily Average SMP using the LR solver dropped from €126.92 to €54.07 while the highest Average SMP from the MIP solver dropped from €123.16 to €59.29.

Consumer Cost Comparison

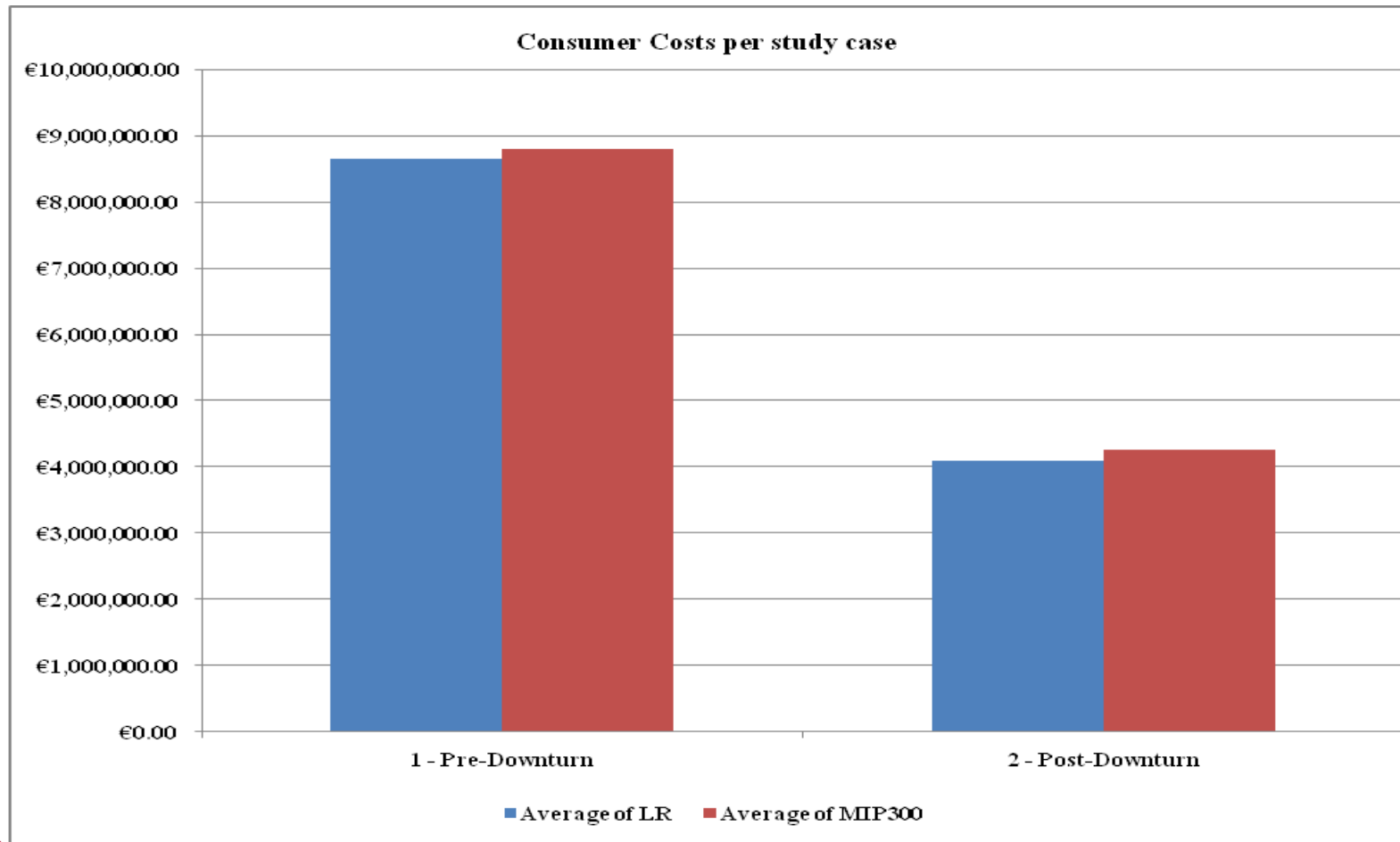
As previously highlighted it is not the aim of the solver to reduce Consumer Costs

- Consumer Costs is calculated as $\sum MSQ * SMP * TPd$ for each Trading Period

The following observations were made:

- In 57% of the study cases, Consumer Costs are increasing with MIP, with an average increase of €500,000 per Trading Day.
- Increases in Consumer Costs are directly related to generator revenues in the SEM which increased by around 2.7% in the solutions from the MIP solver.

Solver Choice in the SEM: LR vs. MIP



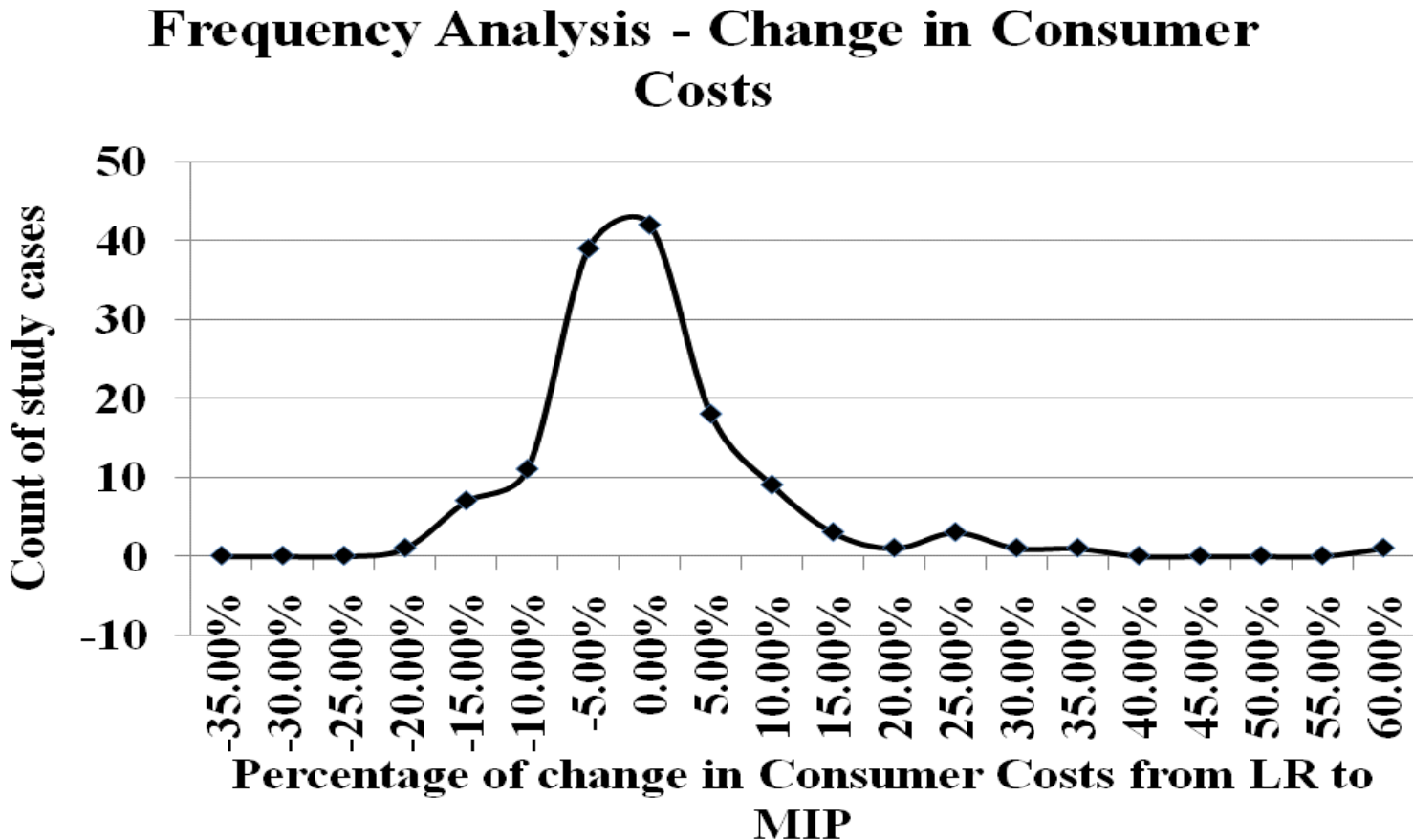
Consumer Cost Comparison

	<i>MIP Decrease</i>	<i>MIP Increase</i>	<i>MIP Decrease as %</i>	<i>MIP Increase as %</i>
MAX	-€1,929,885.96	€2,916,026.72	-16.431%	37.497%
MIN	-€1,450.92	€1,344.13	-0.023%	0.013%
AVERAGE	-€376,862.67	€580,485.18	-4.228%	7.141%
COUNT	50	66		
Total	-€18,843,133.46	€38,312,681.86		

Table 1: Summary of Consumer Cost changes, pre economic downturn

	<i>MIP Decrease</i>	<i>MIP Increase</i>	<i>MIP Decrease as %</i>	<i>MIP Increase as %</i>
MAX	-€571,359.38	€2,247,484.36	-13.787%	60.643%
MIN	-€617.45	€478.88	-0.019%	0.013%
AVERAGE	-€187,434.30	€425,676.27	-4.442%	10.319%
COUNT	16	22		
Total	-€2,998,948.87	€9,364,877.87		

Table 2: Summary of Consumer Cost changes from LR to MIP, post economic downturn



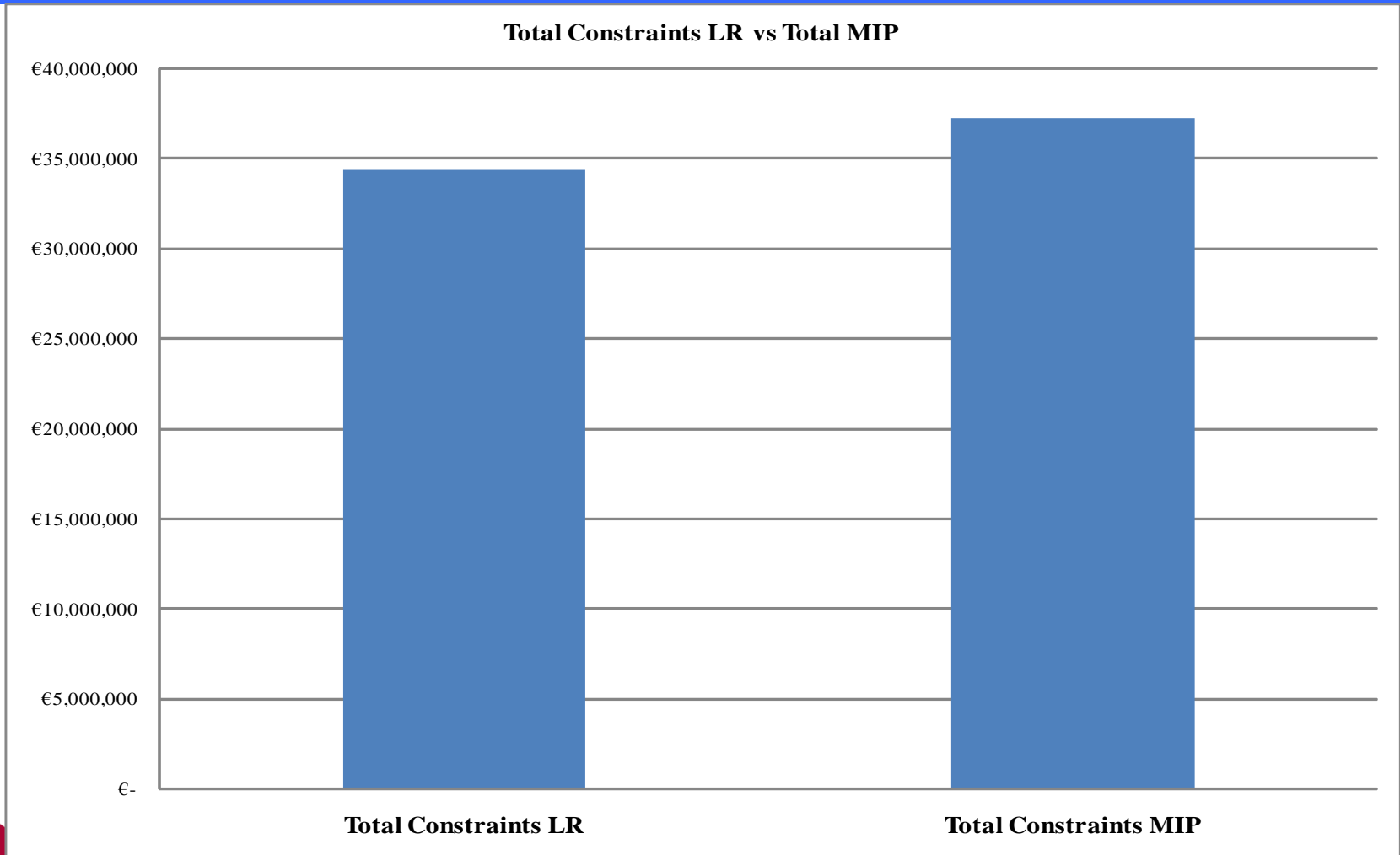
Most of the changes observed are between +/-10% (126 study cases out of 154) with only exceptional cases with extreme variances. +/-10% in monetary terms equal to +/- €700,000.

Constraints Comparison

- Apart from some anomalies throughout the study, MIP solutions incur more expensive Constraints Payments in the majority of cases (121 over 154 total days).
- The LR and MIP solvers mainly produce the same trend in payments; i.e. they both result in payments or they both result in negative payments for generators. There are some anomalies in 5 days that have extreme variances between the LR and MIP solver; however, the majority of variances occur within the +/-30% range.



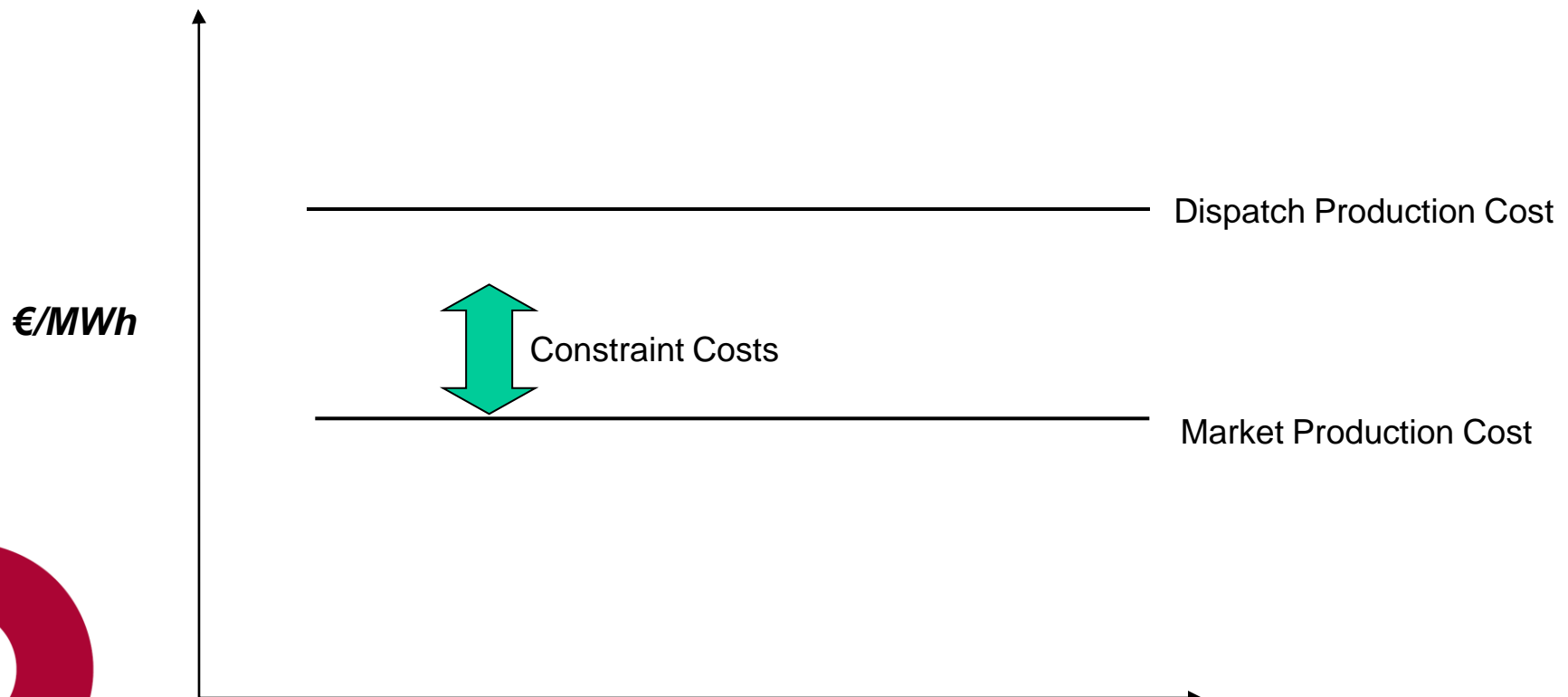
Solver Choice in the SEM: LR vs. MIP



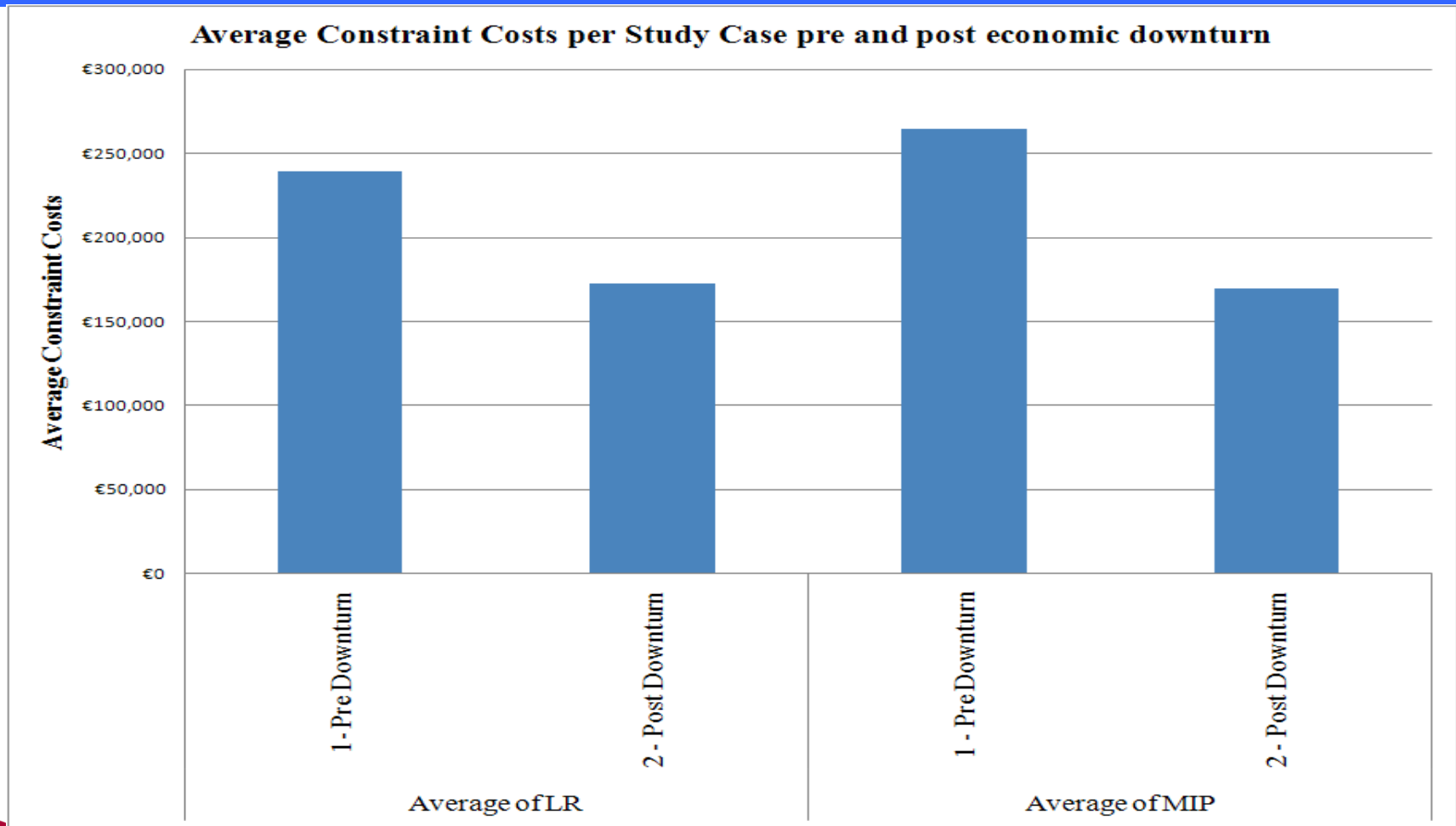
Constraints Costs with LR were over €34M while with MIP were over €37M. The difference between the two solvers equates to an 8% increase with MIP.

Constraints Comparison

- It should be noted that the Dispatch Production Costs did not change, therefore, when the MIP solver reduces MSP Production Costs, an increase Constraint costs is expected.



Solver Choice in the SEM: LR vs. MIP



The pre and post economic downturn average Constraint costs per day with LR dropped from €237,098.85 to €172,562.00. The MIP Constraint costs before the economic downturn were €264,829.73 and dropped afterwards to €169,395.61.

Constraints Comparison

- The system is dispatched based on the output of the RCUC program which uses the MIP solver. It must be remembered that the market program does not contain any Reserve or System Constraints. Therefore, we cannot expect that because we are using the same core MIP solver, that we will get a similar result.
- Also observed the impact of improved Hydro generator scheduling on the net daily totals. The under utilisation of Hydro generators in the schedules from the LR solver are resulting in payments in from thermal generators while no payments are made out to the Hydro generators. This will result in a lower net daily total for Constraints in LR.

Energy Limited Generators Comparison

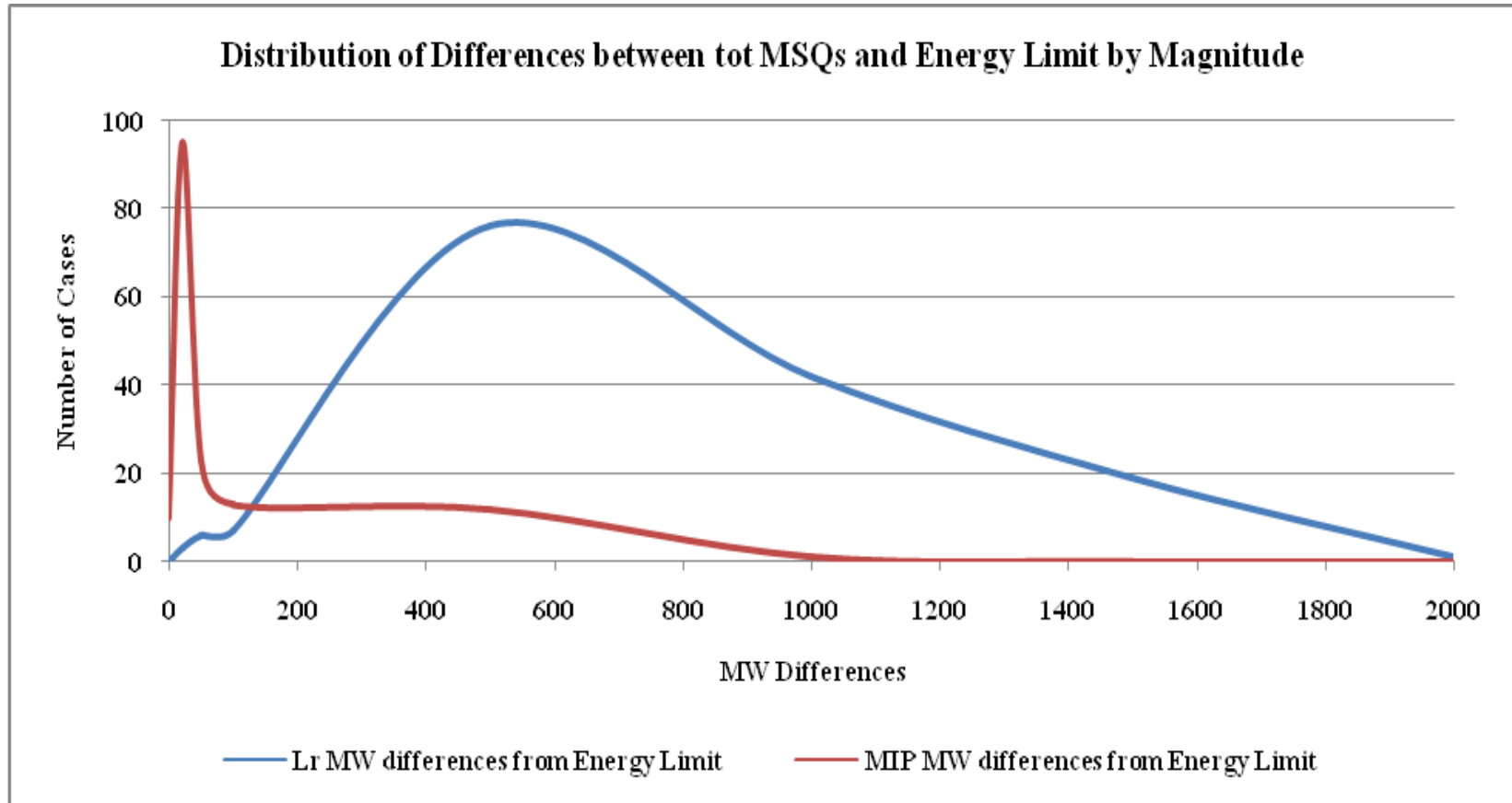
- MIP solver is:
 - Scheduling Hydro units closer to their Energy Limit in all 154 cases studied; and
 - Fully achieving the total Energy Limit in 22 of these cases.

- When using the LR solver, we did not observe the Energy Limits being fully used in any case.

- The average difference between total schedule for Hydro units and Energy Limits amounts to 493MW in LR and just 38MW in MIP.

- The additional utilisation of the Energy Limited Units with the MIP solver occurs during the night valley as the LR solver keeps conventional generators on, while the MIP solver has a more flexible approach and does not refrain from switching generator off and on again.

Solver Choice in the SEM: LR vs. MIP



Key Observations

- In over **83%** of study cases there was a decrease, generally within 1%, in MSP Production Cost with the MIP solver.
- In **82.4%** of cases LR and MIP PC are within $-/+1%$ of each other
- The MIP solver produced higher Average System Marginal Prices and Consumer Costs in over **57%** of cases. This was despite an observed reduction in peak prices. Consumer Costs and Market Prices are not primary objectives of the solver.
- Constraint Costs were also observed to increase with MIP; however, as the Dispatch Production Cost is a static value across all studies, when one solver reduces the MSP Production Cost this will lead to increases of this nature.

Key Observations cont'd

- Increased use of Energy Limited or Hydro stations in **ALL** runs using the MIP solver.
- Other fuel and station technology types did not appear to be impacted by the solver choice.
- The comparison between MIP and LR is a comparison between two sub-optimal solutions.

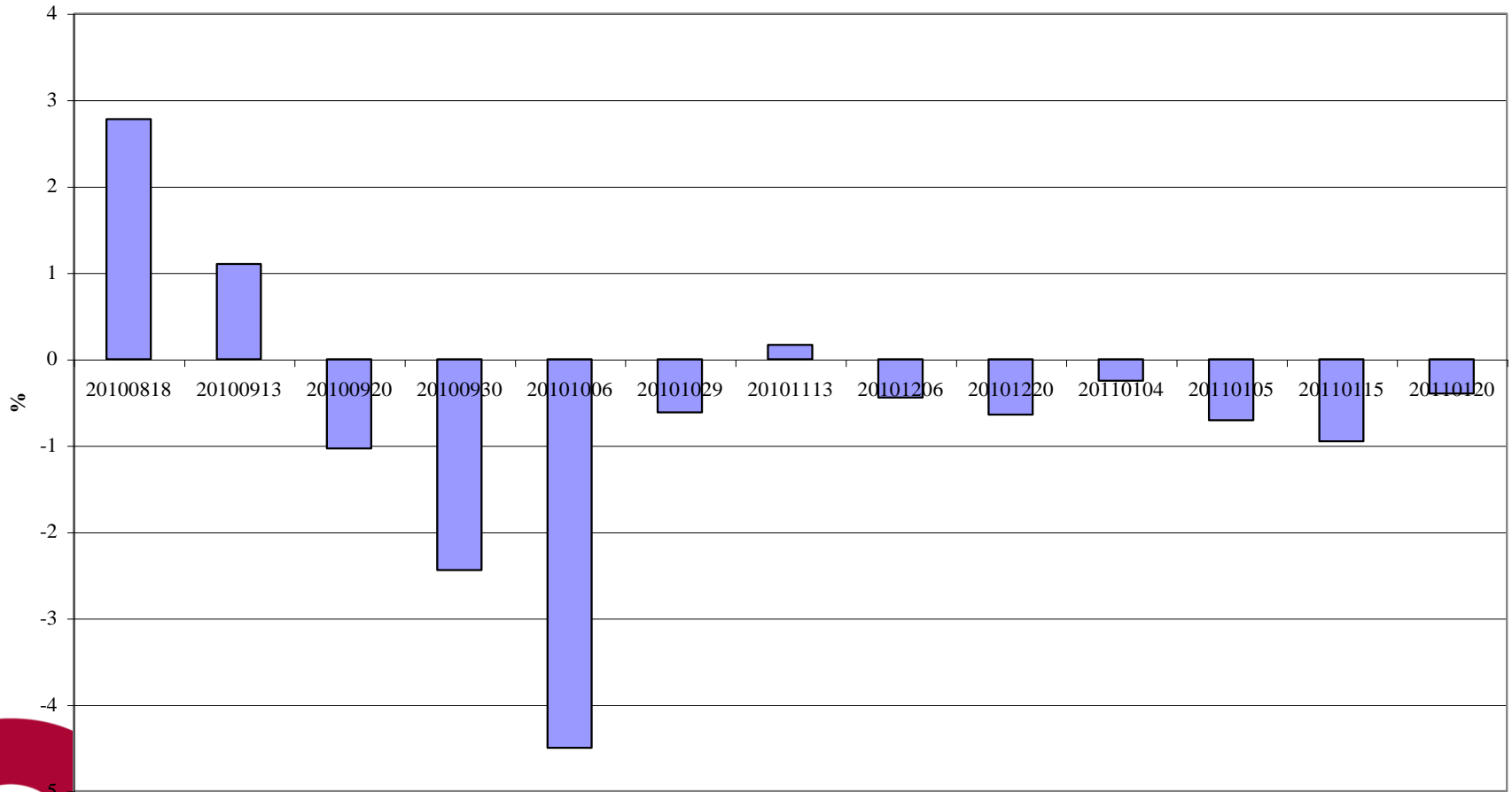
**A look at some up to date observations
from live Market Operations runs**



Solver Choice in the SEM: LR vs. MIP

MIP v LR Comparison August 2010 to Jan 2011

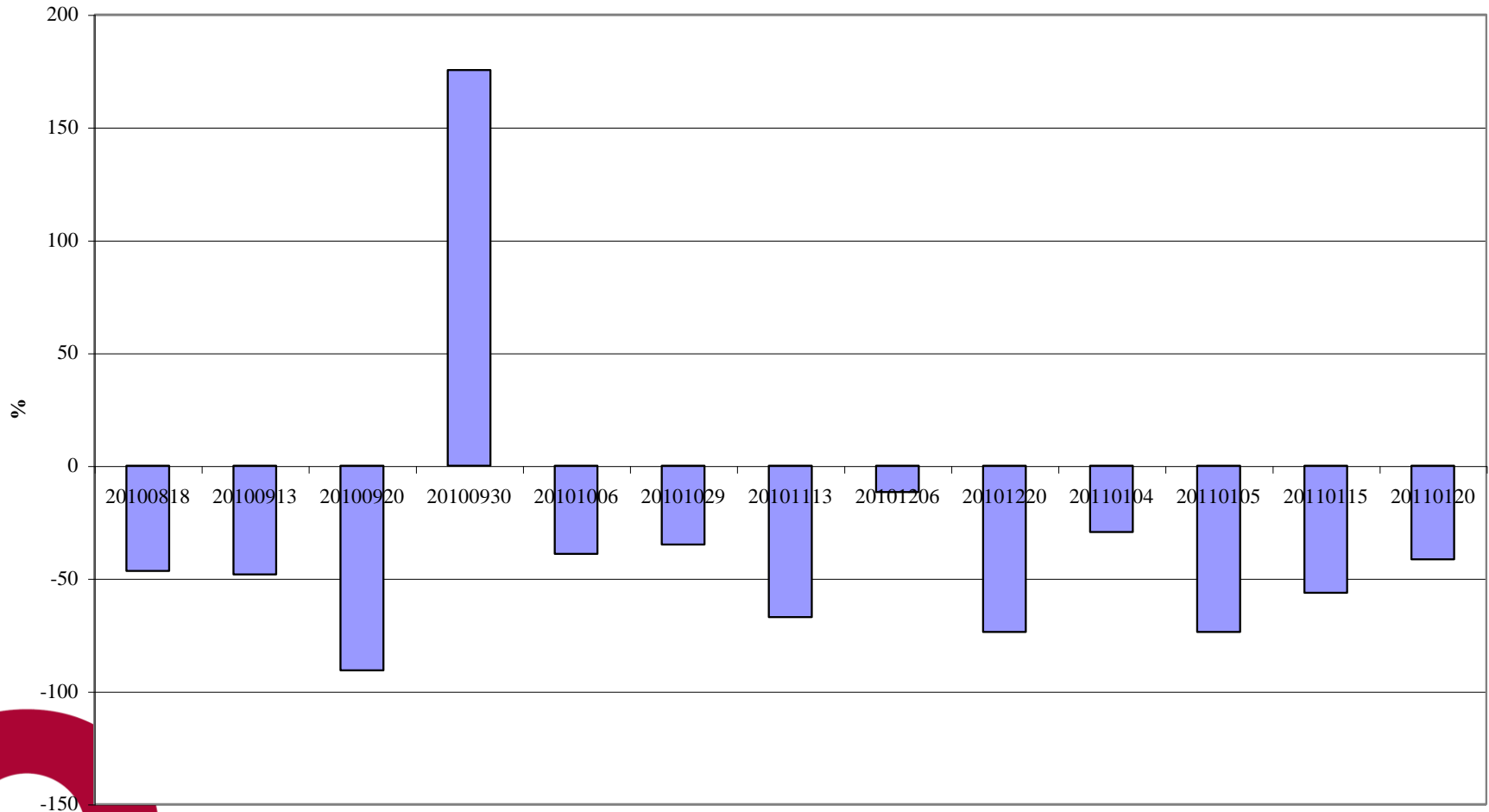
■ Production Cost



MIP Production Cost lower in 10 out of 13 EP2 runs carried out

MIP v LR Comparison August 2010 to Jan 2011

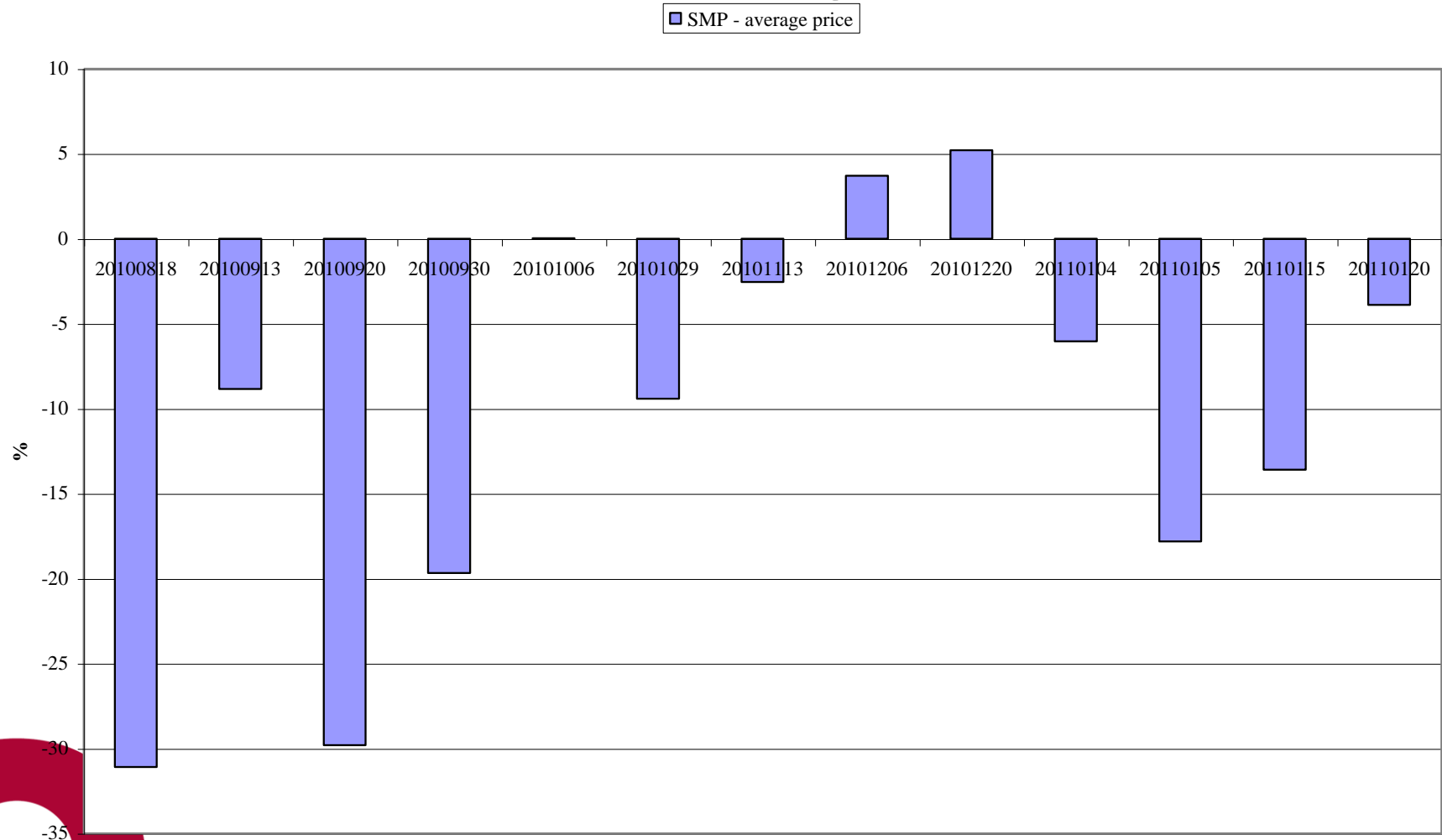
■ SMP - price spikes



MIP reduced price spikes lower in 12 out of 13 EP2 runs carried out

Solver Choice in the SEM: LR vs. MIP

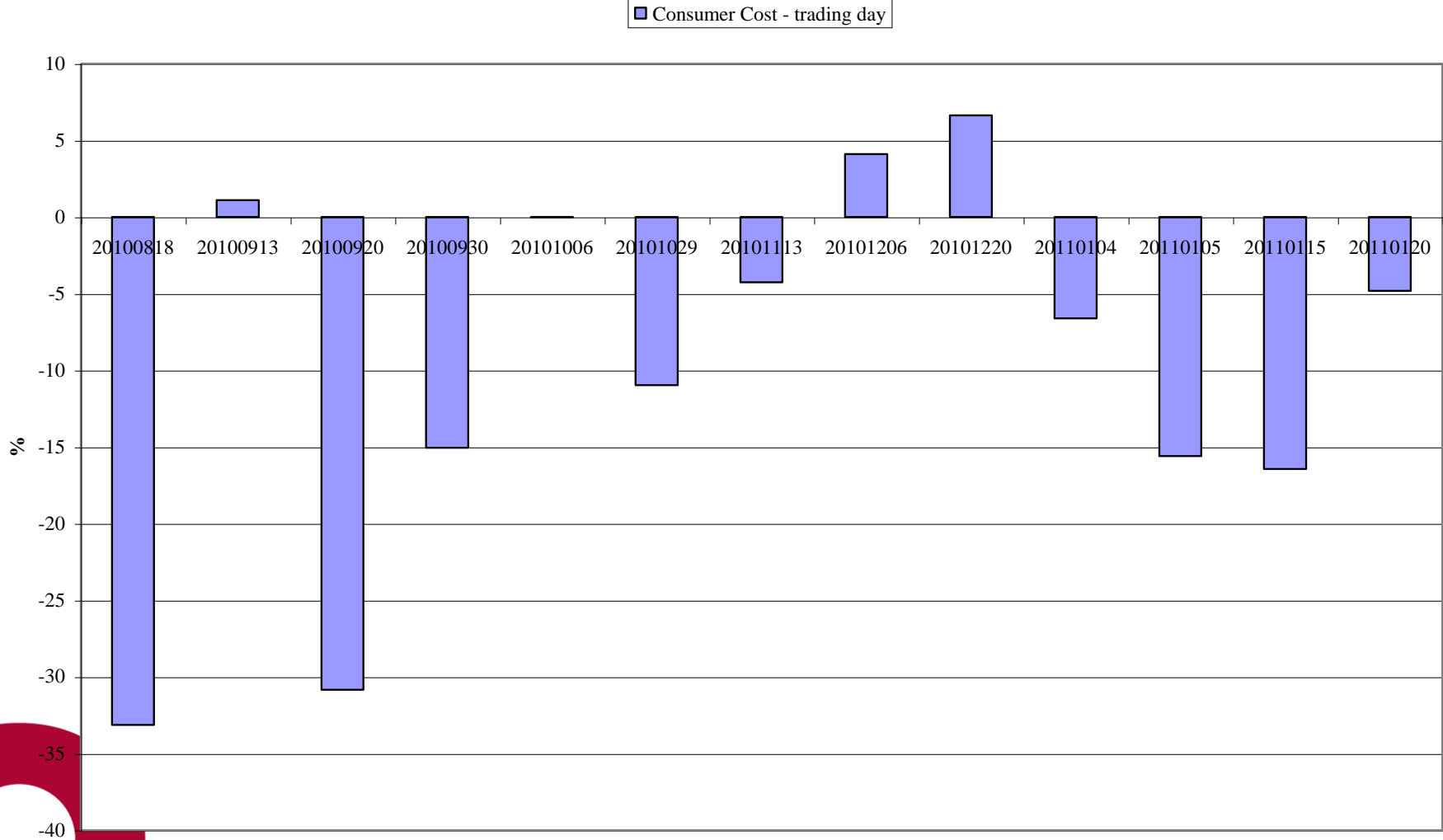
MIP v LR Comparison August 2010 to Jan 2011



MIP reduced average SMP in 10 out of 13 EP2 runs carried out

Solver Choice in the SEM: LR vs. MIP

MIP v LR Comparison August 2010 to Jan 2011



MIP reduced average Consumer Cost in 10 out of 13 EP2 runs carried out

MIP v LR Comparison August 2010 to Jan 2011 – EP1

Similarly over 17 EP1 runs carried out:

- Using MIP over LR reduced MSP Production Costs in 15 cases;
- Using MIP over LR reduced Price Spikes in 16 cases;
- Using MIP over LR reduced Average SMP in 11 cases;
- Using MIP over LR reduced Consumer Costs in 12 cases;

Summary

In summary the detailed observations are as follows –

- Using MIP over LR will lead to reduced MSP Production Costs;
- Using MIP over LR has been observed to make better use of Energy Limited Generators;
- Using MIP with the five minutes timeout setting provides best return;
- There is no guarantee that MIP schedules will produce higher or lower SMPs, Consumer Costs or Constraints Costs;

What next?



- Change to MIP as the Solver of Choice?
 - Current Solver of Choice for PJM and California;
 - Ontario moving to MIP;
 - New York are evaluating MIP;
 - Speed and reliability are key drivers for change;

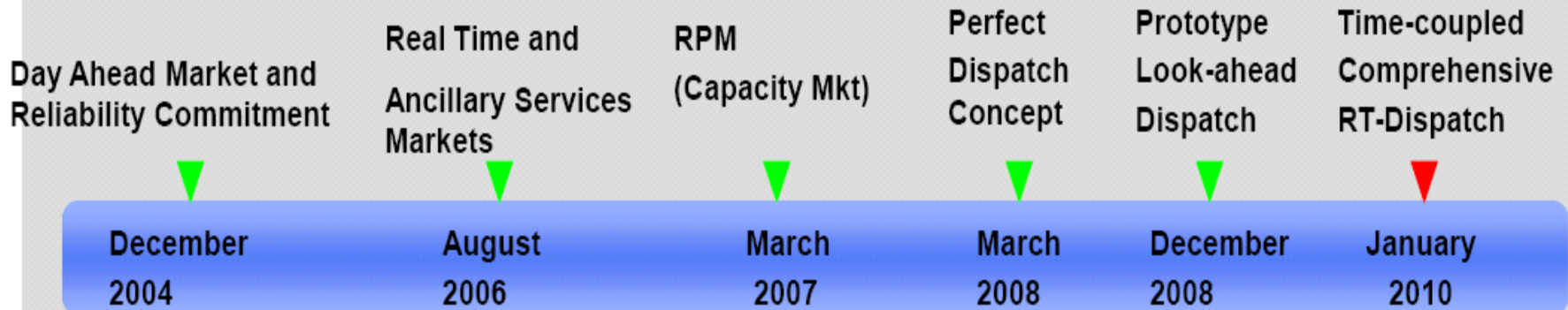
- Benefits of MIP Implementation, Andrew Ott, Senior Vice President, PJM
 - Global optimality;
 - More accurate solution;
 - Improved modelling of security constraints;
 - Enhanced resource modelling capability;
 - a) Generation;
 - b) Demand response;
 - c) Transmission Devices;
 - More adaptable problem definition;

Source – <http://www.ferc.gov/eventcalendar/Files/20100601131610-Ott,%20PJM.pdf>

Solver Choice in the SEM: LR vs. MIP

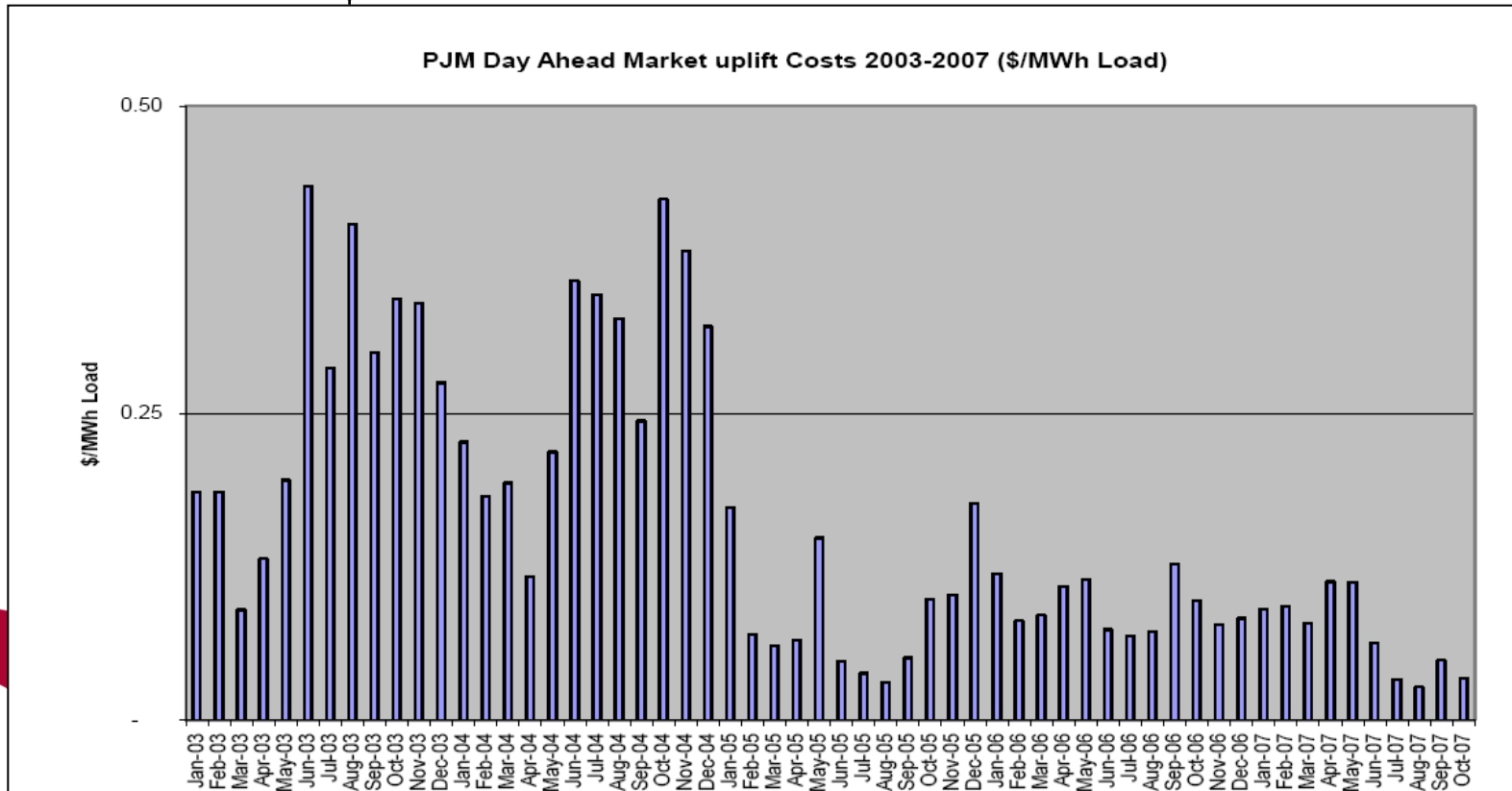
- PJM exclusively uses MIP in every situation where Linear Programming cannot be used

Timeline of MIP Implementation in Production systems



Solver Choice in the SEM: LR vs. MIP

- PJM Uplift Costs below – MIP phased in since 2004
- PJM saw uplift benefits drop from approx. \$US 0.25/MWh- \$US 0.30/MWh to \$US 0.08/MWh-\$US 0.12/MWh



- Should Run Time be 300 or 600 seconds
 - Process review would be required to establish which is more practical;
 - However, SEM-O believes the impact on current process of increasing the run time may be minimal;
 - Of the 49 cases that did not solve to within the Optimality Gap, only 6 of these solved to that level with MIP600;
 - Further analysis may be required to establish the preferred run time;
 - PJM have seen software improvements during longer running times;

- LR as a back up solver?
 - Are there any reasons why we would need LR as a back up?
 - Would pricing re-runs be done in the Solver of Choice?
 - Would there be financial implications here in terms of supporting both solvers?
 - E.g. PJM do not maintain both solvers as it is deemed to expensive to support

➤ Parallel Running

- Periods of 2-6 months appear to be the norm in terms of running both solvers in parallel
- Ontario anticipate 3 month timeframe when they switch from LR to MIP
- PJM ran parallel for 6 months when changing from LR to MIP
- Criteria would be set to compare the daily LR and MIP runs

- Parallel Running
 - For example comparisons could be made between:
 - Stability of commitment patterns
 - Total Production Costs
 - Optimality gap
 - Incidence of software failure
 - SEM-O would need to review resources for Pricing and Scheduling area

- If LR is chosen to remain the Solver of Choice will its shortcomings be addressed?
 - ALTCOM – currently being addressed by vendor;
 - Single Ramp Rate – currently modification proposal;
 - Treatment of Hydro generators in UUC;

- Financial implications

- Ontario market found that MIP would be easier and cheaper to modify
- Estimates of MIP being 3 times cheaper than LR to change
- E.g. if the Single Ramp Rate is to be addressed DSI, UUC & Altcom would have to be changed in LR whereas with MIP, only DSI would have to be amended

Thank you for your attention.

Questions?

